Generic closed curve

\underline{unsigned Gauss code}

Gauss 1840
Tait 1870s

Dehn 1923

\[ \text{abcdefgchaigjkhbiFejk} \]

\[ \text{Smoothing} \]

\[ \text{Alexande numbering} \]
\[ \text{wind}(Y_1 \times) = \]
\[ \# \text{ccw cycles containing } x \]
\[ - \# \text{cw cycles containing } x \]

rotation \# = \# ccw cycles
- \# cw cycles

Seifert decomposition
Gauss' parity condition

Substring between two occurrences of any symbol has even length

Proof via winding number

⇒ Any two generic closed curves generically intersect in even # points

⇒ abab is not a planar Gauss code

abcd cdecdebe abcabcdecode abcadedede

Gauss

Nagy graph

Tait
reverse every other edge of image graph

Parity $\Rightarrow$ Nagy graph of any curve is Eulerian

every node has in degree 2 and out degree 2

Dehn code = sequence of self-tangencies

Pick an Euler tour of Nagy graph

Dehn code = sequence of vertices

"Dehn diagram"
Dehn: Gauss code can be realized by generic planar curve

Dehn code can be realized by planar self-touching / weakly simple curve

Dehn diagram is \textit{planar} $\rightarrow$ inside arcs outside arcs

Baum-Zwiebel Figur

co.onion-onion
tree-cotree decomposition
interlacement graph

Lemma: Dehn diagram is planar if and only if its interlacement graph is bipartite.

Rosenstiehl + Tarjan: O(n) time
Planarity testing "left-right"