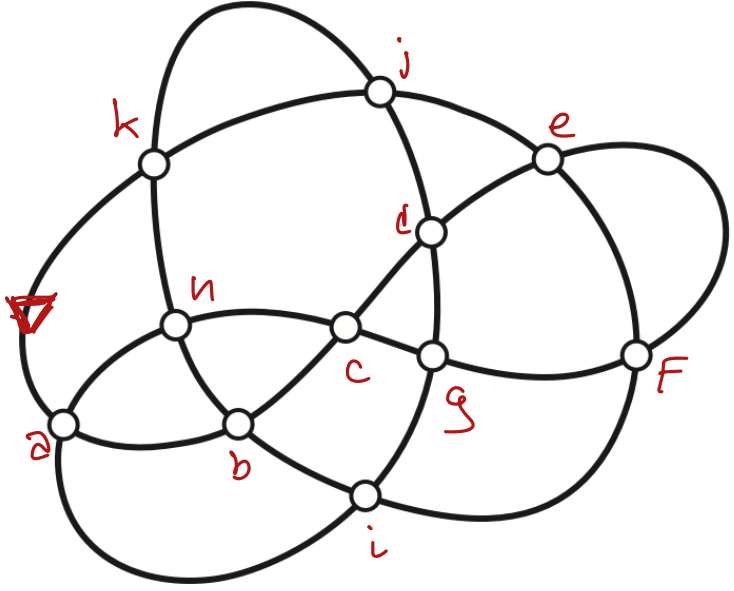
Generic closed curve

unsigned Gauss code

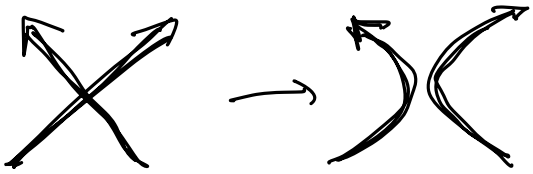
Gauss 1840
Tait 1870s

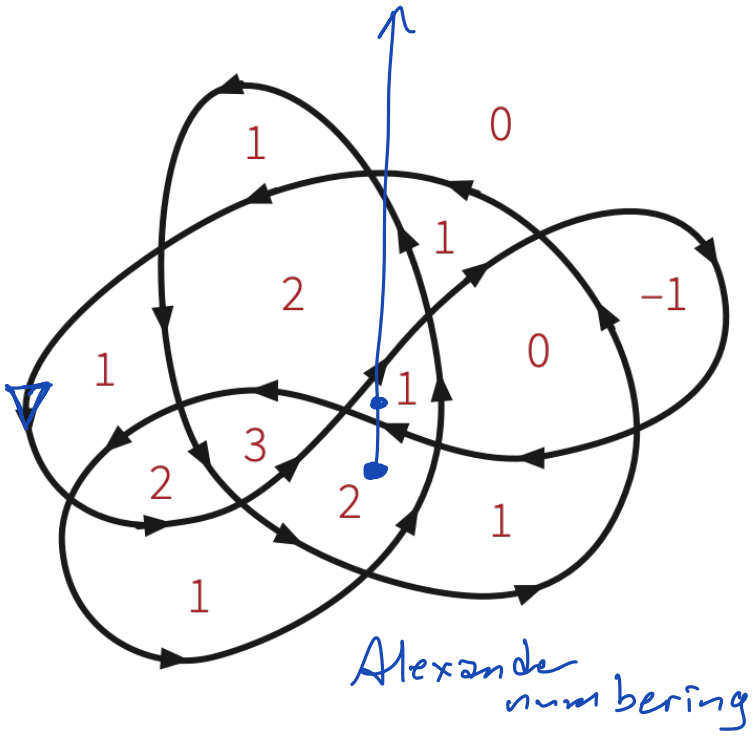
Dehn 1923



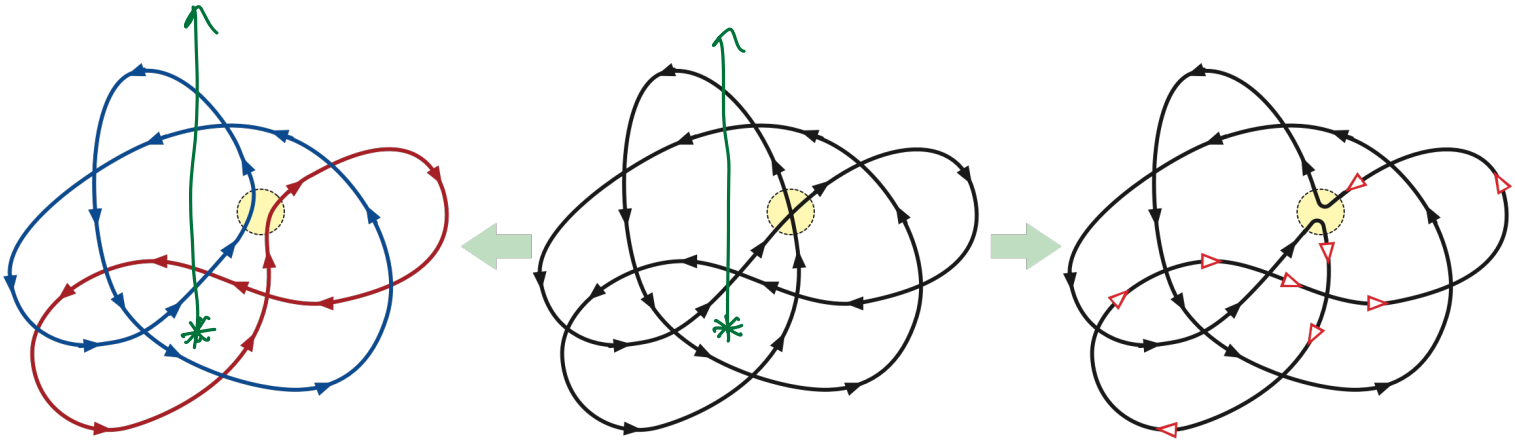
abcdefgchaigdjkhbiFejk

Smoothing





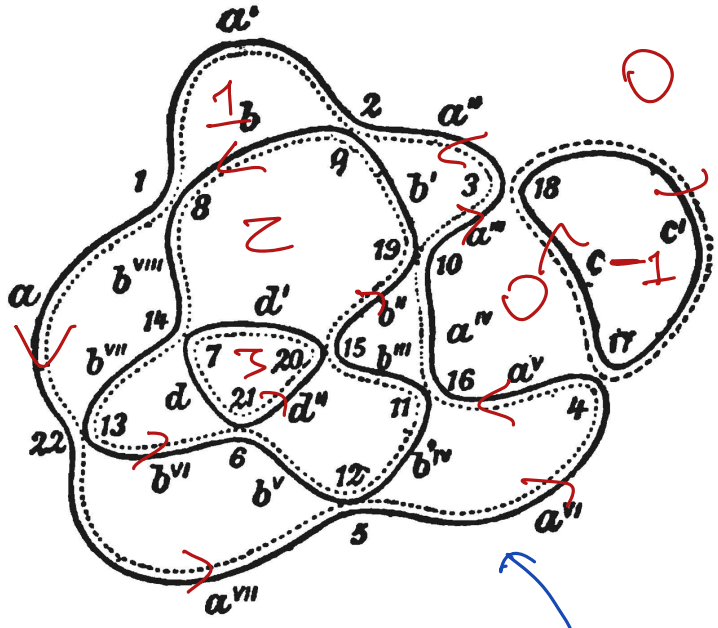
Alexander numbering

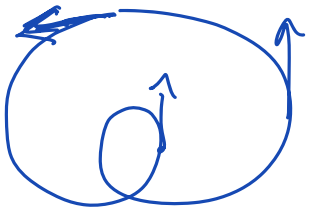


keep orientation
breaking connection

keep connection
reverses orientation

wind(γ, x) =
ccw cycles containing x
− # cw cycles containing x

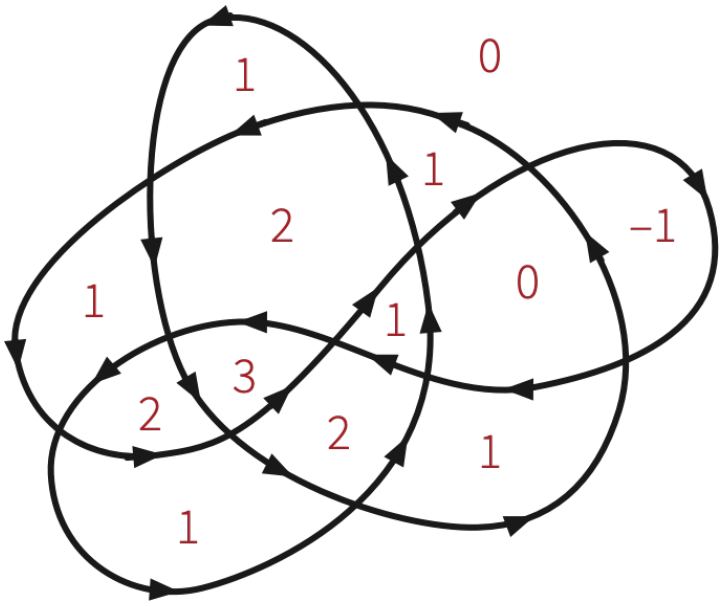




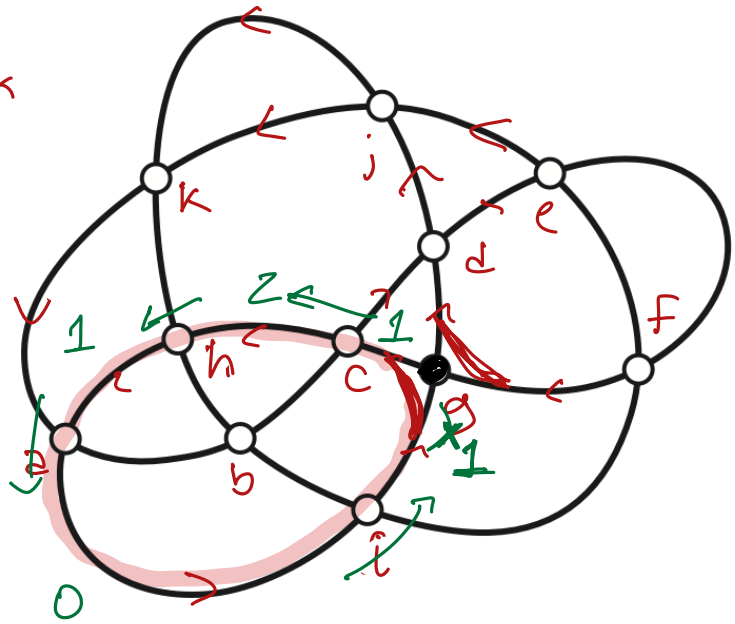
rot # 2

rotation # = # ccw cycles
− # cw cycles

Seifert decomposition


1
0
1
2
−1
1
0
1
3
2
2
1
1

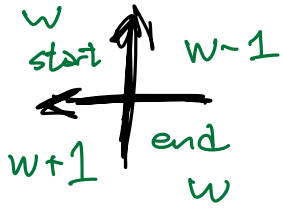
abcdefgchaigdjkhbiFejk



Gauss' parity condition

substring between two occurrences of any symbol has even length

Proof via winding #s

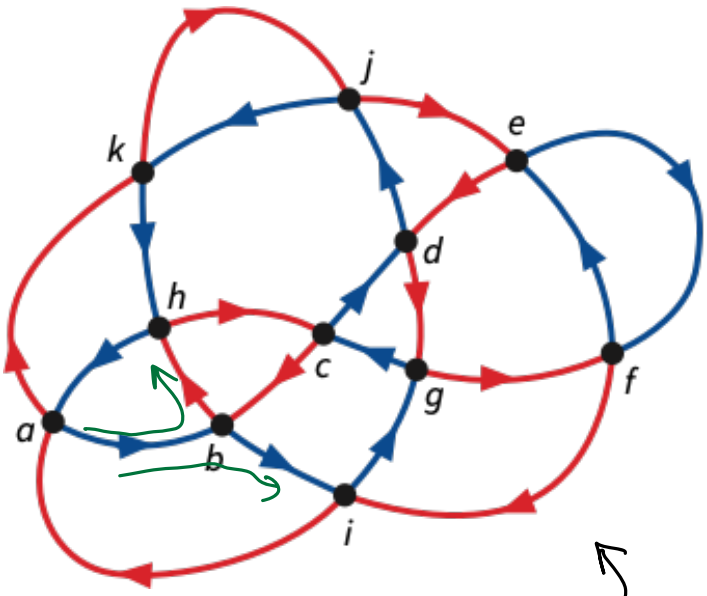


$\Rightarrow$ Any two generic closed curves generically intersect in even # points

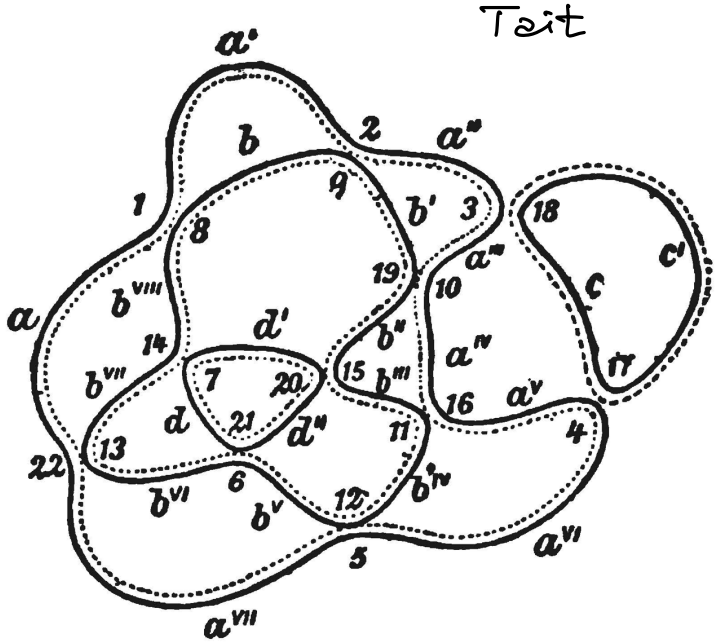
$\Rightarrow$ abab is <u>not</u> a planar Gauss code

abcadcedbe abcdbdecde abcadebdec

Gauss Tait



Nagy graph



reverse every other edge of image graph

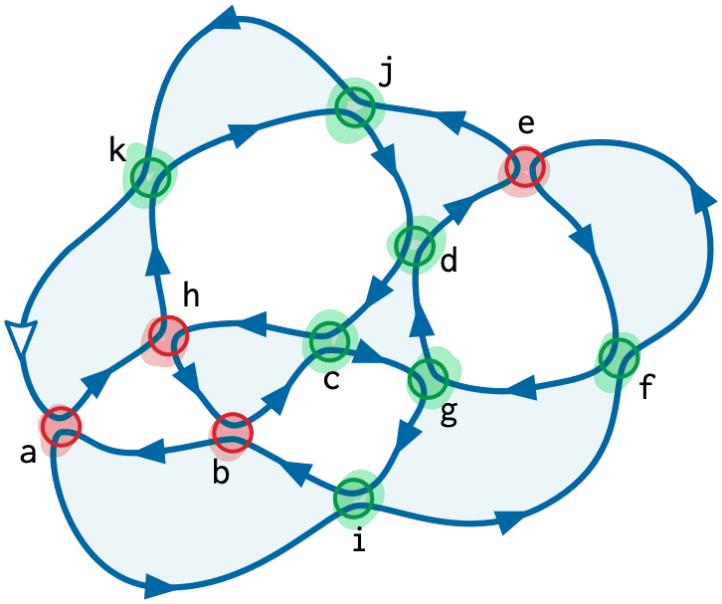
abcdefgcha igdjkhbiFejk

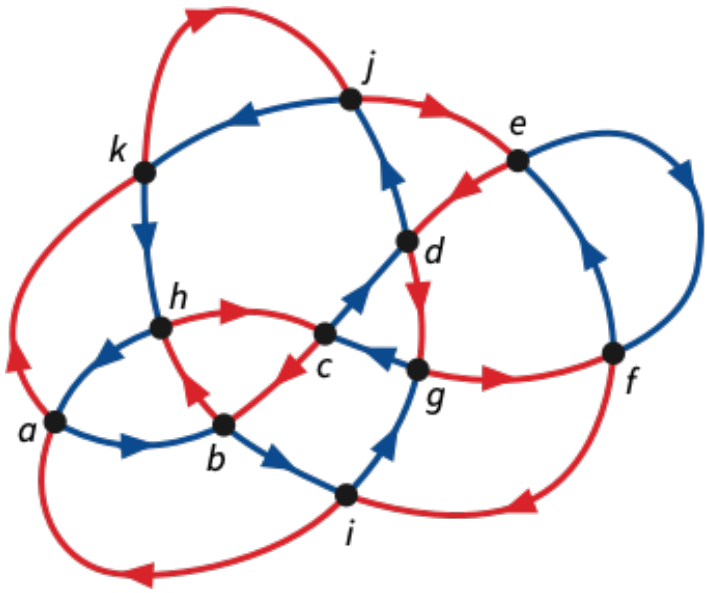
$\leftarrow a \rightarrow b \leftarrow c \rightarrow d \leftarrow e \rightarrow f \leftarrow g \rightarrow c \leftarrow h \rightarrow a \leftarrow i \cdots$

Parity $\Rightarrow$ Nagy graph of any curve is <u>Eulerian</u>

$\Downarrow$

every node has indegree 2
outdegree 2





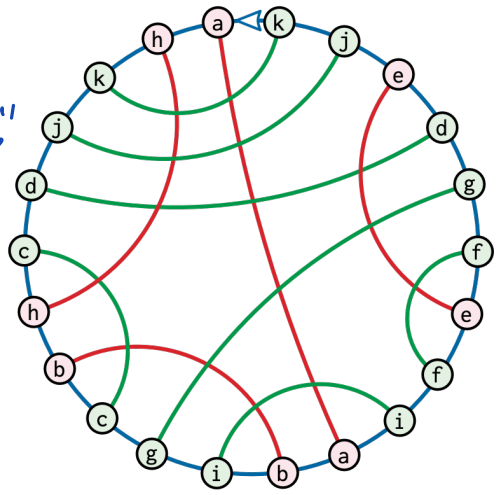
ahkjdch....

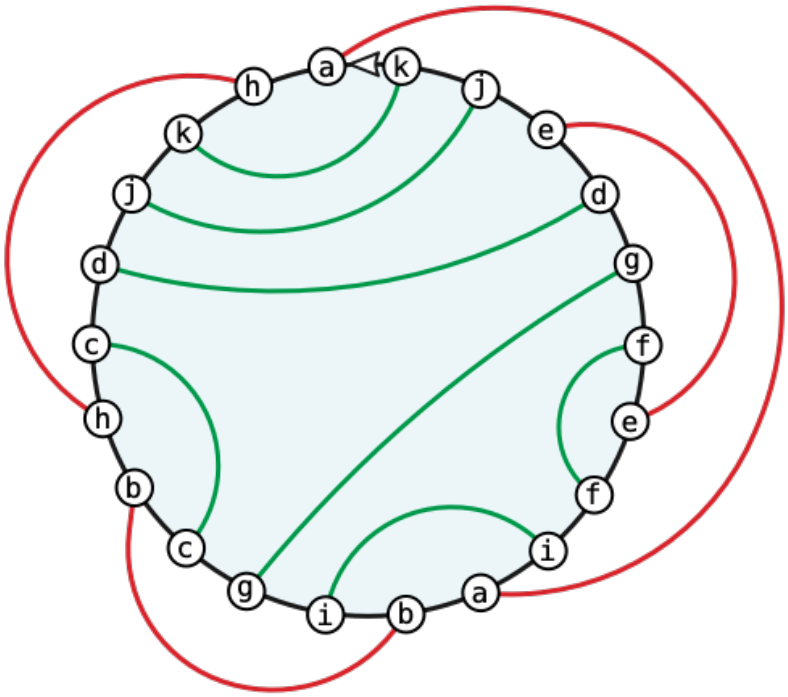
<u>Dehn</u> code = sequence of self-tangencies

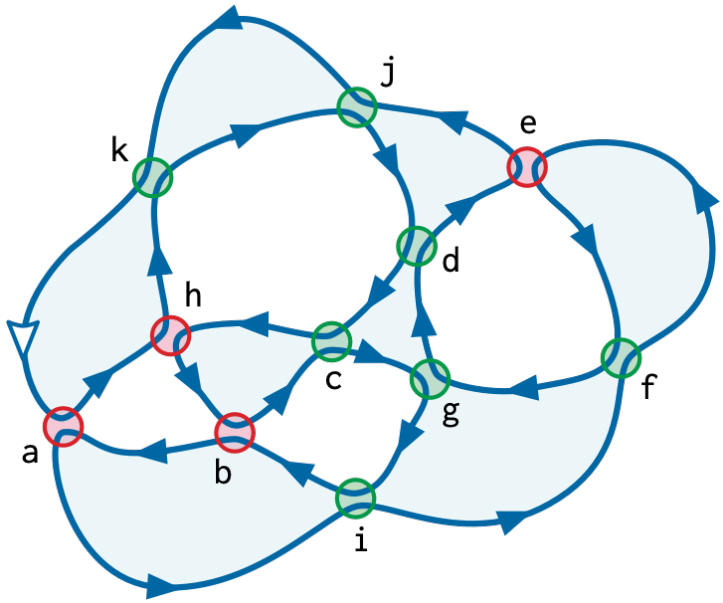
Pick an Euler tour of Nagy graph

<u>Dehn code</u> = sequence of vertices

"<u>Dehn diagram</u>"







<u>Dehn</u>: Gauss code can be realized by generic planar curve

$\Updownarrow$

Dehn code can be realized by planar self-touching / weakly simple curve

$\Updownarrow$

Dehn diagram is <u>planar</u> $\rightarrow$ inside arcs / outside arcs

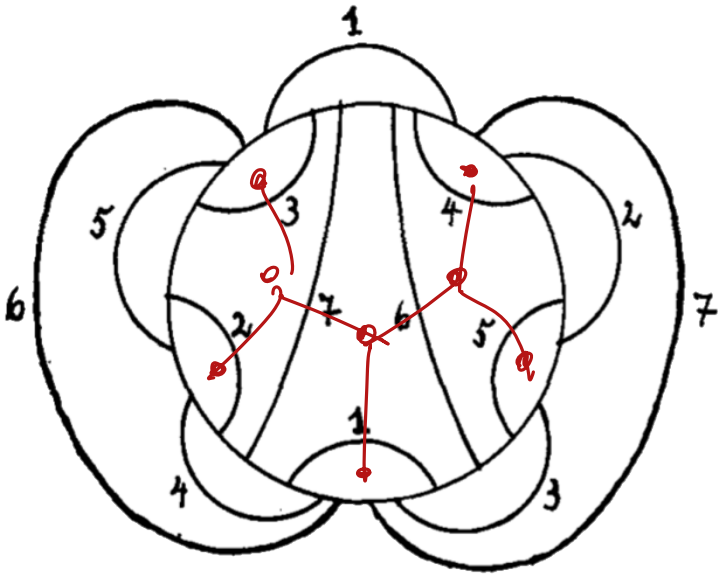


Fig. 15.

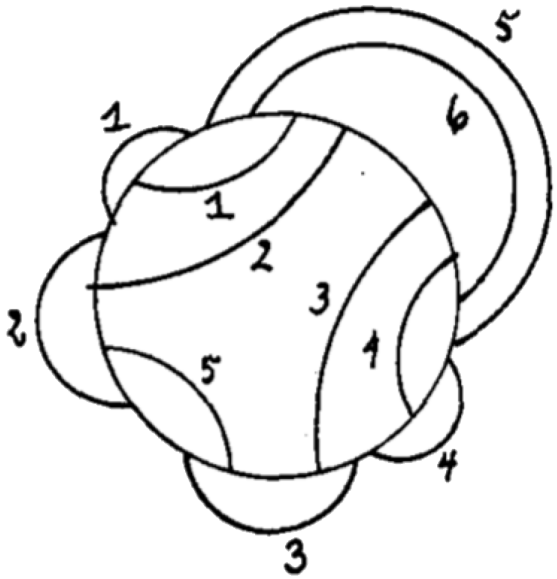


Fig. 14.

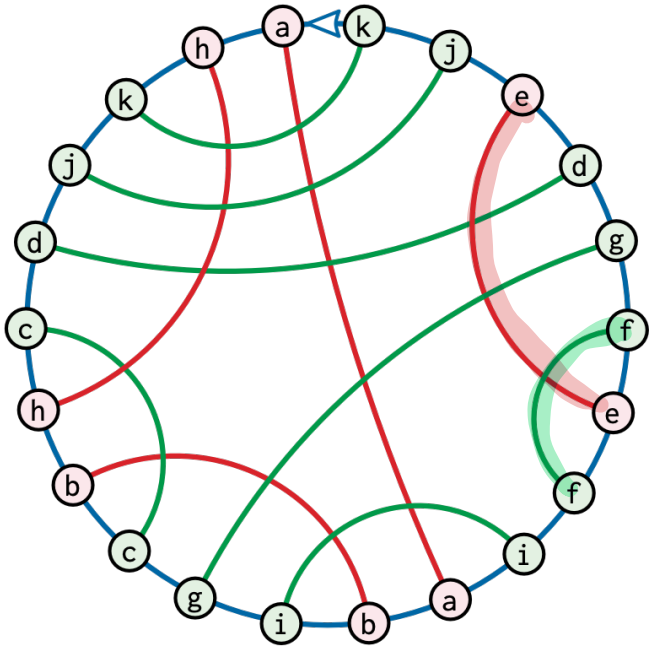
Baum-Zwiebel Figur

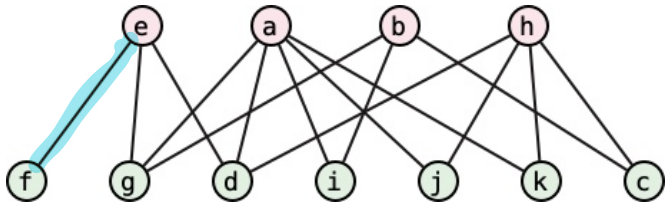
coonion-onion

tree-cotree decomposition

e...f...e...f

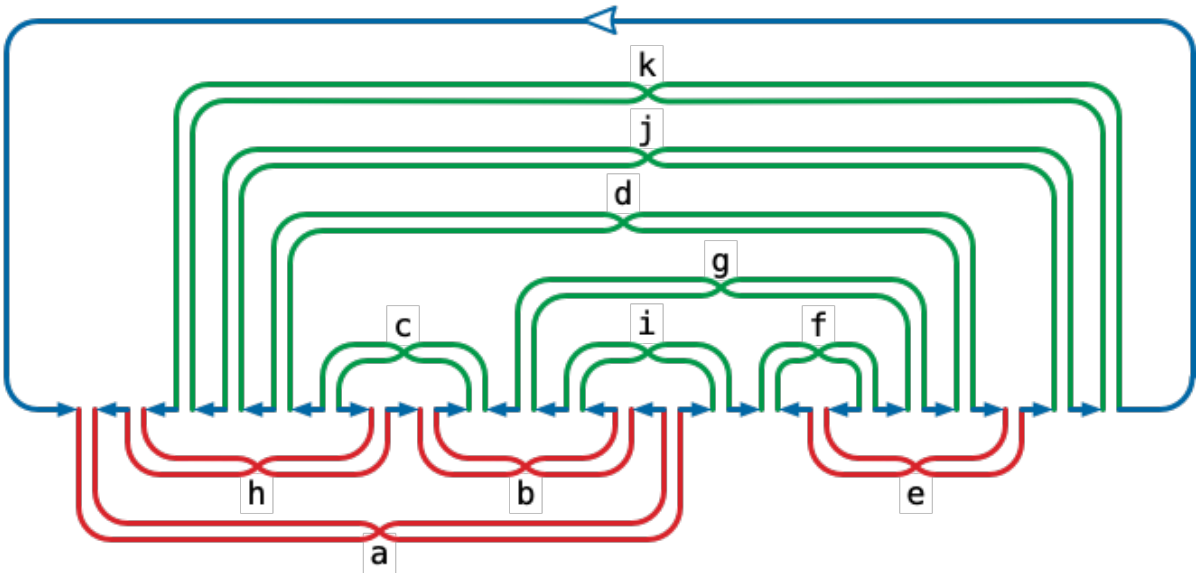
interlacement graph





Lemma: Dehn diagram is planar iff its interlacement graph is bipartite.

Rosenstiehl + Tarjan: $O(n)$ time



Planarity testing "left-right"