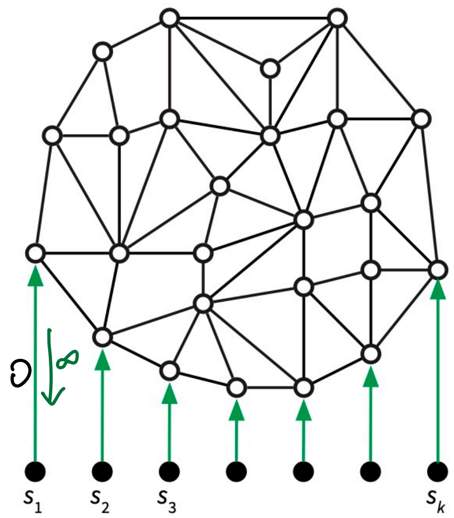
Multiple source shortest paths

Implicitly compute distance from every s_i
to every vertex v.

Preprocess Σ $\rightarrow$ $O(S(n) \log n)$ time

Query $\rightarrow$ $O(\log n)$

shortest path tree
$O(n \log n)$ Dijkstra
$O(n)$ Henzinger et al



$\boxed{\text{SIMPLE}}$ Planar map Σ

- $\ell(u \to v) \neq \ell(v \to u)$
- source vertices $s_1 \cdots s_n$ in order on outer face
- $\Sigma \setminus S$ strongly connected
- shortest paths unique

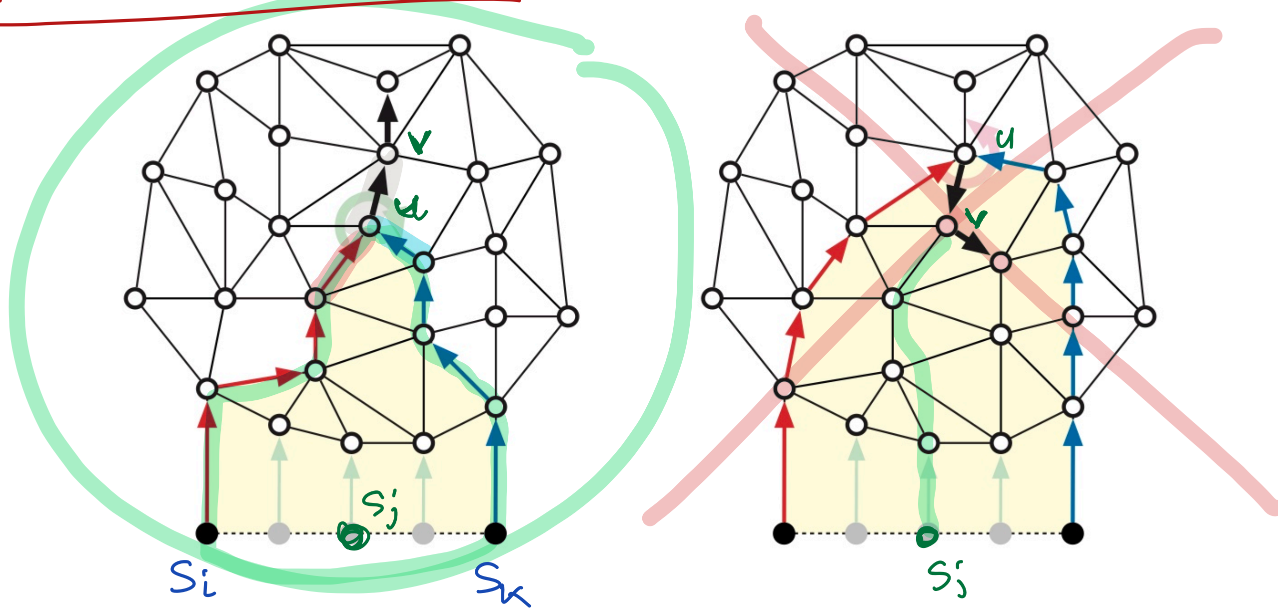
Das Kipouridis Probst Gutenberg Wulff-Nilsen SOSA 2022

Divide-and-Conquer

H = minor of Σ

MSSP-Prep(H, i, k)
  $H' \leftarrow$ Filter(H, i, k)
  $j \leftarrow \lfloor \frac{i+k}{2} \rfloor$
  MSSP-Prep(H', i, j)
  MSSP-Prep(H', j, k)

Filter(H, i, k)



T_j = sh. path tree rooted at s_j

① Compute T_i and T_k ($O(S(n))$ time)

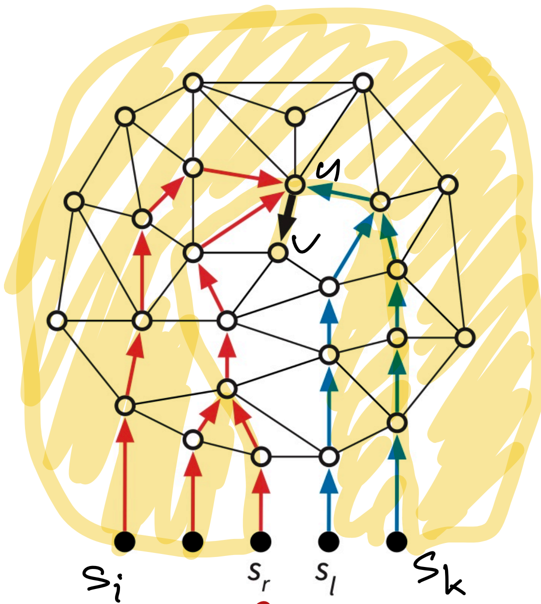
$u \to v$ is <u>properly shared</u> by T_i and T_k

- → $pred_i(v) = pred_k(v) = u$
- if $pred_i(u) = pred_k(u)$, then $pred_i(u) \to u$ is prop. shared
- otherwise

  $pred_i(u)$, v, $pred_k(u)$ are ordered clockwise around u.

→ $u \to v$ is an edge in T_j for <u>all</u> $i \le j \le k$.

$u \to v$ is shared by all T_j's but not prop. shared by T_i and T_j



Contraction

$rep(v) \leftarrow u$ $\quad$ $off(v) \leftarrow \ell(u \to v)$

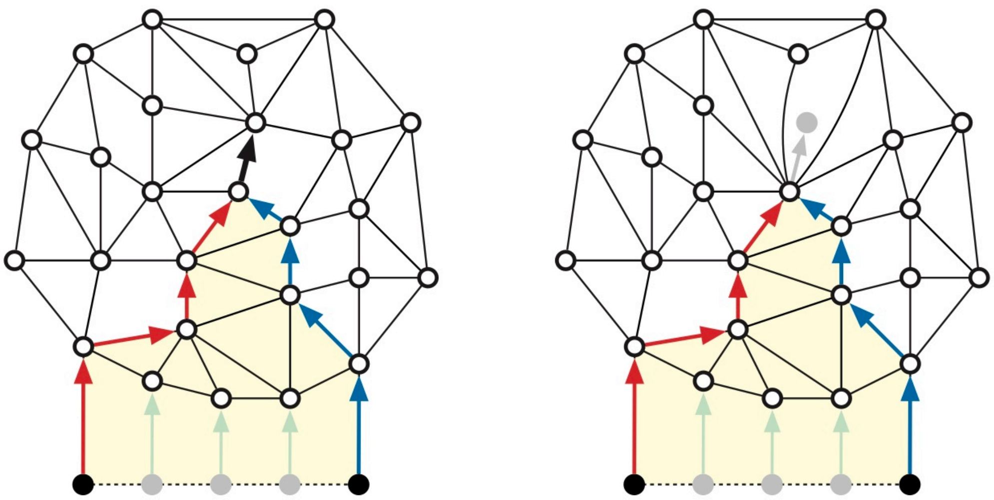
For every $w \to v$, set $\ell \leftarrow \infty$

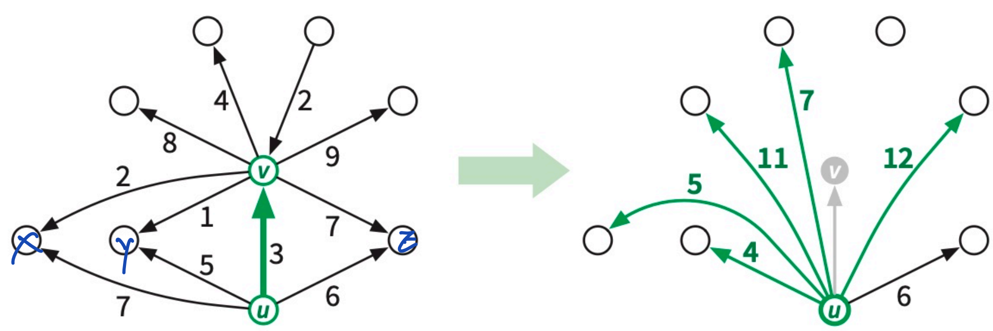
for every $v \to w$ $\quad$ $\ell(v \to w) \leftarrow \ell(v \to w) + \ell(u \to v)$

if $pred_{i,k}(w) = v$, then $pred_{i,k}(w) \leftarrow u$

resolve parallel edges at end

Contract uv



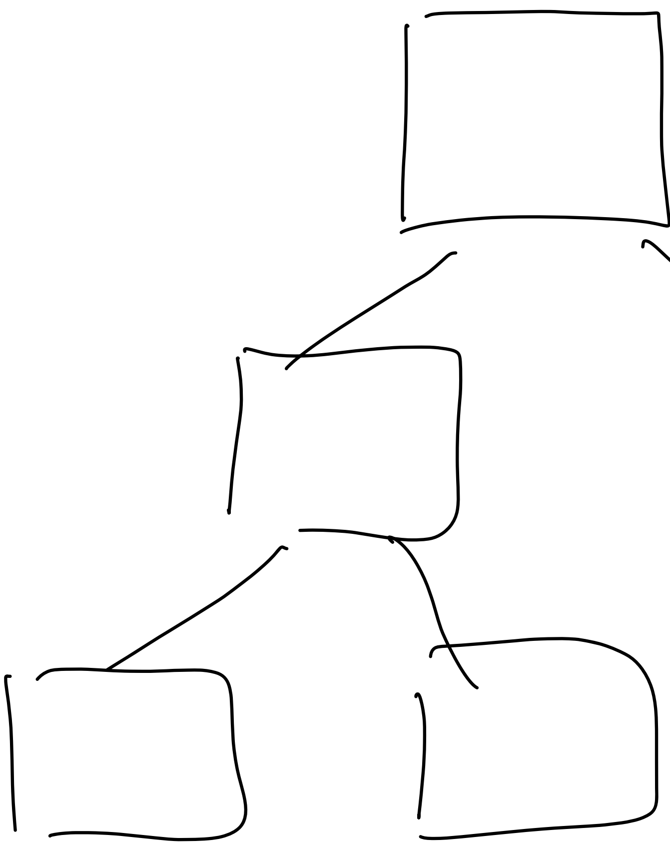


Discover + contract properly shared edges via preorder traversal of T_i

Key Invariant:

$$\boxed{dist_j(w) = dist'_j(rep(w)) + off(w)}$$

Filter takes $O(S(u))$ time



Rec

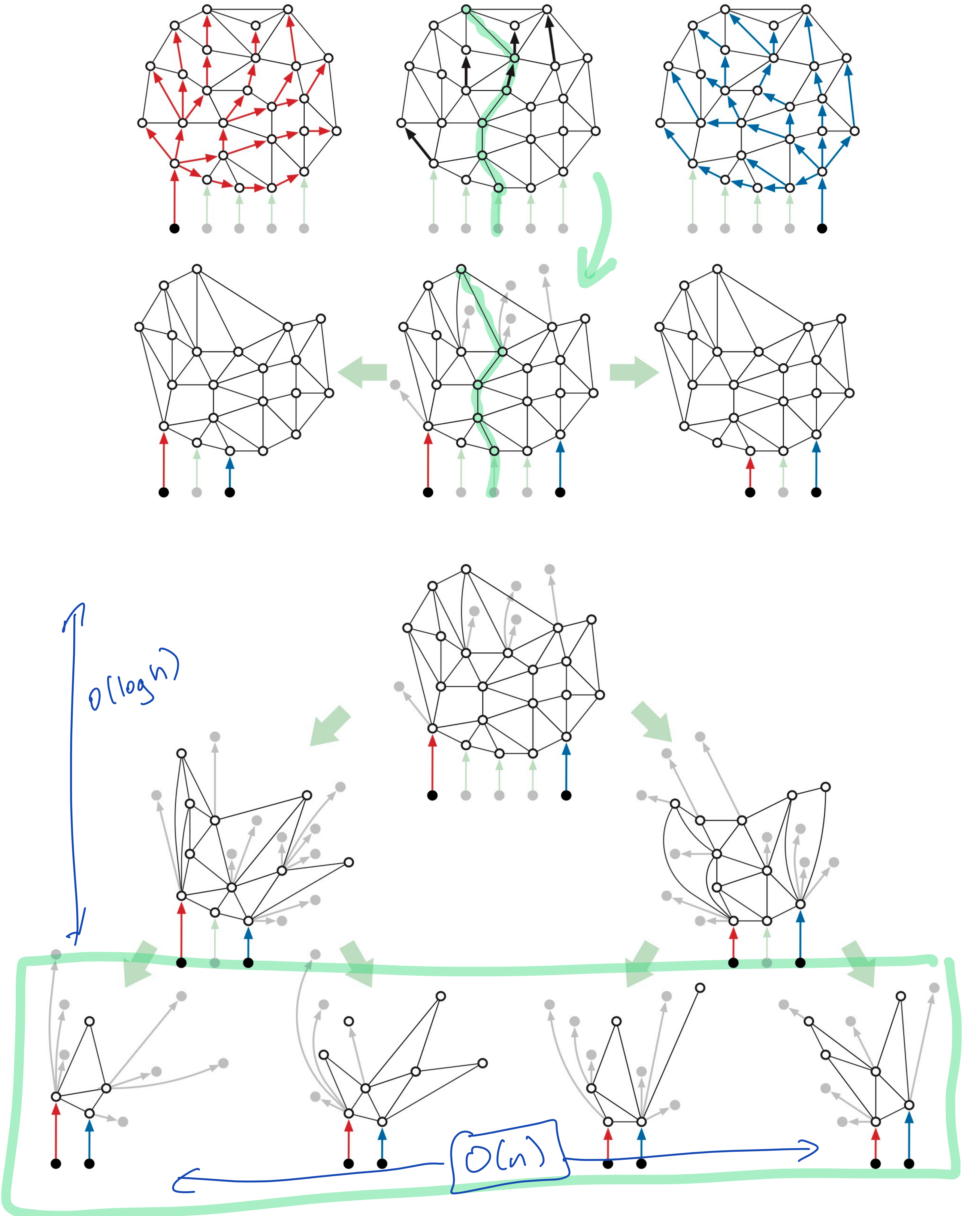
i, k
for every vertex v of H
$dist_i(v)$
$dist_k(v)$
$rep(v)$
$off(v)$

Query(Rec, j, v) $\quad$ $dist(s_j, v)$ in Rec.H

$O(\log h)$

if j = Rec.i return Rec.$dist_i(v)$
if j = Re.k return Rec.$dist_k(v)$
if $j \leq$ Rec.left.k.
return Query(Rec.left, j, Rec.off[v]) + Rec.off(v)

else — right —


O(log n)
O(n)

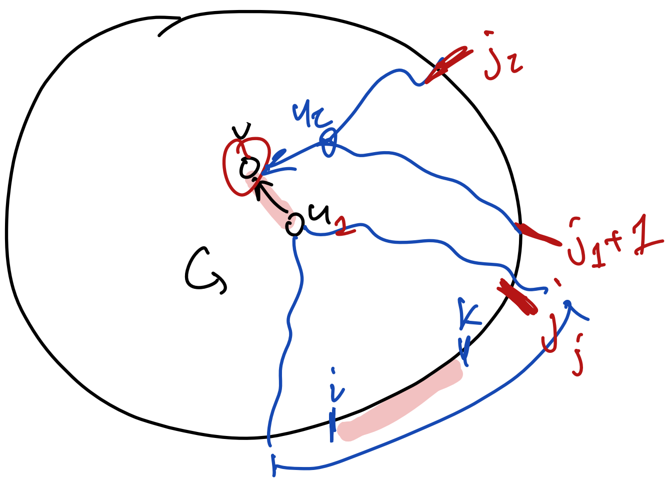
Analyis key claim:

Sum of #vertices represented at Rec's at any level of the tree is $O(n)$

Contracting one properly edge doesn't create or destroy other properly shared edges.

$\text{Filter}(H, i, k) = \text{Filter}(\Sigma, i, k)$

"properly shared" is wlog wrt Σ



Index j is interesting for v if $\text{pred}_j(v) \to v$ is NOT prop. shared by T_j + T_{j+1}

(1) v is stored in Rec iff j is interesting to v where $\text{Rec.parent}.i \le j \le \text{Rec.parent}.k$

(2) Each vertex v has $\le \deg(v)$ interesting j.

$\Rightarrow$ At any level, v is stored in $\le 2 \cdot \deg(v)$ Recs.

$$\sum_v 2 \cdot \deg(v) = 4 \cdot E \le 4(3n-6) < 12n$$