Projects Groups of up to 3 (or 4)

- Theoretical
- Experimental
- Application

} 10–15 pages

- Scholarly 20–25
- Visualization
- Creative

+ 20 minutes

Proposal — individual 2-3 pages

Faster Shortest Paths

Dijkstra: $O(m + n\log n) = O(n \log n)$ time

Bellman-Ford: $O(mn) = O(n^2)$ time

Separators: $O(\sqrt{n})$ vertices that split G into components each with $\leq 3n/4$ vertices

Cycle separators: simple cycle C with $O(\sqrt{n})$ vertices...

each compnt $\Sigma \setminus C$ has $\leq 3n/4$ vertices.

Good r-division: Partition of faces of Σ into $O(n/r)$ pieces

- $O(r)$ vertices
- $O(\sqrt{r})$ bdry vertices
- $O(1)$ holes

$O(n \log r)$ time
↓
$O(n)$

Dense distance graph [Fakcharoenphol Rao '03]

For each piece R in a good r-division $\mathcal{R}$

Let $X(R)$ = weighted clique over boundary vertices R

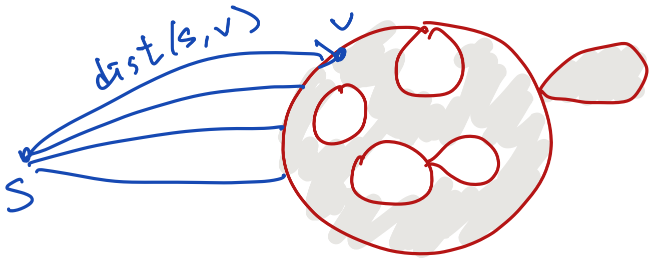
$\ell(u \to v) = dist_R(u, v)$

DDG has $O(n/\sqrt{r})$ vertices + $O(n)$ edges

Let's beat Dijkstra!

1. Build a good r-division
2. Build DDG — run MSSP around every hole of each piece

parametric



$O(r \log r)$ time per piece

$\Downarrow$

$O(n \log r)$ total.

3. Run Dijkstra on DDG $\quad O\left(n + \frac{n}{\sqrt{r}} \log n\right)$

4. Fill in each piece via Dijkstra $\quad O\left(\frac{n}{r}\right) \cdot O(r \log r)$

$r = \log^2 n \implies O(n \log\log n)$

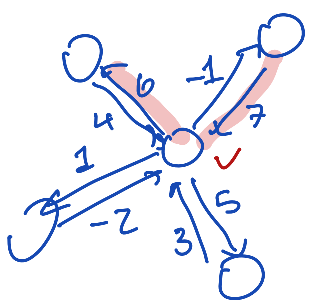
$\rightarrow O(n)$ more work

Henzinger Klein Rao Subramanian

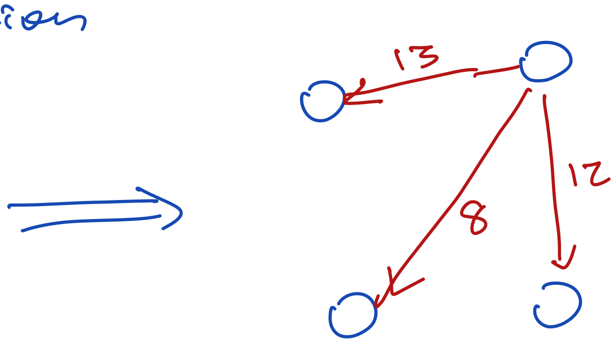
Beat Bellman-Ford $\longrightarrow O(n \log^2 n / \log\log n)$ time

Lipton Rose Tarjan — generalized nested dissection $O(n^{3/2})$

vertex elimination



star-mesh transform



$\ell(u \to w) \Leftarrow \ell(u \to v) + \ell(v \to w)$

or

$\ell(u \to w) \leftarrow \min\{\ell(u \to w), \ell(u \to v) + \ell(v \to w)\}$

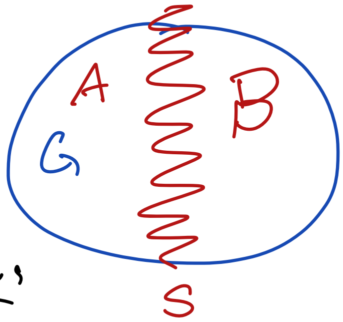
eliminate $v \leftarrow O(\deg^2(v))$

Recursive compute $\mathrm{dist}(s,u)$ for all $u \neq v$

$$\mathrm{dist}(v) \leftarrow \min_u \{\mathrm{dist}(u) + \ell(u \to v)\} \qquad O(\deg(v))$$

Different elimination orders $\Rightarrow$ diff time

LRT:



1. Recursively order A
2. Recursively order B
3. order S arbitrarily

Assume good r-div's

$T_\downarrow(r)$ = elimination time

$T_\uparrow(r)$ = recovery time

for one piece of size r

AA	0	SA
0	BB	SB
AS	BS	SS

$$T_\downarrow(r) \leq T_\downarrow(r_1) + T_\downarrow(r_2) + O(r^{3/2})$$

$$\Rightarrow O(r^{3/2})$$

$r_1 + r_2 \leq r$

$r_1, r_2 \leq \frac{3r}{4}$

$$T_\uparrow(r) \leq T_\uparrow(r_1) + T_\uparrow(r_2) + O(r)$$

$$\Rightarrow O(r \log r)$$

$\Rightarrow$ Tutte drawing in $O(n^{\omega/2})$ time

Repricing Graph G edge/dart lengths $\ell(d)$

Price function $\pi : V \to \mathbb{R}$

$$\ell'(u \to v) = \pi(u) + \ell(u \to v) - \pi(v)$$

$$\ell'(u \rightsquigarrow v) = \pi(u) + \ell(u \rightsquigarrow v) - \pi(v)$$

Same shortest paths!

$$\boxed{\pi(v) = \mathrm{dist}(s,v)}$$

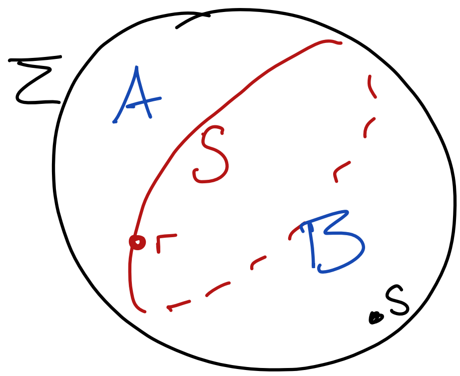
$\ell'(u\to v) = dist(s,u) + \ell(u\to v) - dist(s,v) \geq 0$

Mehlhorn Schmidt 1983 ÷

Goal: $dist_\Sigma(s,\Sigma)$

Σ – simple triangulation weighted darts (maybe < 0)

s – source vertex



① Balanced cycle sep. S

Recursively compute $dist_A(r, A)$

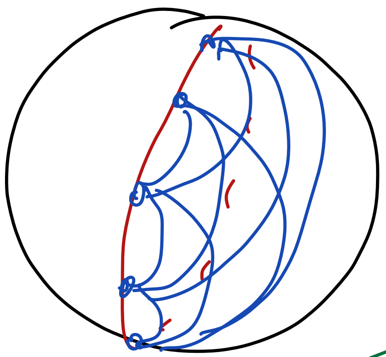
$dist_B(r, B)$

② Compute $dist_A(S,S)$ and $dist_B(S,S)$

~~reprice A~~

~~$\pi(v) = dist_A(r,v)$~~

MSSP



③ Compute $dist_\Sigma(S,S)$

Bellman Ford $\to O(n^{3/2})$

$O(n\alpha(n))$ via Monge / SMAWK

4,5 Cleanup post processing