Projects

Groups of up to 3 (or 4)

- Theoretical
- Experimental
- Application
- Scholarly
- Visualization
- Creative

Proposal - individual 2-3 pages

Faster Shortest Paths

Dijkstra: $O(|V| + |E| \log |V|) = O(|V| \log |V|)$ time
Bellman-Ford: $O(|V||E|) = O(n^2)$ time

Separators: $O(|V|)$ vertices that split $G$ into components each with $\leq \frac{3}{4} |V|$ vertices

Cycle separators: simple cycle $C$ with $O(|V|)$ vertices...

each component $G \setminus C$ has $\leq \frac{3}{4} |V|$ vertices.

Good $r$-division: Partition of faces of $G$ into

$O(n \log r)$ time

$O(n)$ pieces
- $O(r)$ vertices
- $O(\sqrt{r})$ boundary vertices
- $O(1)$ holes

Dense distance graph [Fakechaveenphol Zoo '08]

For each piece $R$ in a good $r$-division $R$

Let $X(R) = \text{weighted clique over boundary vertices } R$

$d(u,v) = \text{dist}_{R}(u,v)$
Let's beat Dijkstra!

1. Build a good r-division
2. Build DDG — run MSSP around every hole of each piece
   \[ \text{time per piece} = O(r \log r) \]
   \[ \text{total} = O(n \log r) \]
3. Run Dijkstra on DDG
4. Fill in each piece via Dijkstra

\[ r = \log^2 n \Rightarrow O(n \log \log n) \]

Beat Bellman-Ford — \( O(n \log^2 n / \log \log n) \) time

Lipton Rose Tarjan — generalized nested dissection \( O(n^{3/2}) \)

Vertex elimination

Star-mesh transform

\[ l(u, v) = \min \{ l(u, w) + l(w, v), l(u, w) \} \]
eliminate $v \leq O(\deg^2(v))$

Recursive compute $\text{dist}(s,v)$ for all $u \neq v$

$\text{dist}(v) \leq \min_{u} \text{dist}(u) + l(u \rightarrow v) \leq O(\deg(v))$

Different elimination orders $\Rightarrow$ different time

LRT:

Assume good $r$-division:

$T_{\downarrow}(r) = \text{elimination time}$

$T_{\uparrow}(r) = \text{recovery time}$

for one piece of size $r$

$T_{\downarrow}(r) \leq T_{\downarrow}(r_1) + T_{\downarrow}(r_2) + O(r^{3/2})$

$\Rightarrow O(r^{3/2})$

$r_1 + r_2 \leq r$

$T_{\uparrow}(r) \leq T_{\uparrow}(r_1) + T_{\uparrow}(r_2) + O(r)$

$\Rightarrow O(r \log r)$

$\Rightarrow$ Tutte drawing in $O(n^{(\omega/2)})$ time

Repricing: Graph $G$ edge/arc lengths $l(l(d))$

Price function $\Pi : V \rightarrow \mathbb{R}$

$l'(u \rightarrow v) = \Pi(u) + l(u \rightarrow v) - \Pi(v)$

$l'(u \rightarrow v) = \Pi(u) + l(u \rightarrow v) - \Pi(v)$

Same shortest paths!

$\Pi(v) = \text{dist}(s,v)$
Mehlhorn Schmidt 1983

Goal:
\[ \text{dist}_S(s, \Sigma) \]
\[ s \text{- source vertex} \]

1. Balanced cycle sep. \( S \)
   - Recursively compute \( \text{dist}_A(r, A) \) and \( \text{dist}_B(r, B) \)

2. Compute \( \text{dist}_A(S, S) \) and \( \text{dist}_B(S, S) \)
   - Reprice \( A \)
   - \( \Pi(v) = \text{dist}_A(r, v) \)
   - \( \text{MSSP} \)

3. Compute \( \text{dist}_\Sigma(S, S) \)
   - \( \text{BellmanFord} \rightarrow O(n^{3/2}) \)

4.5 Cleanup post-processing

\[ O(n a(n)) \text{ via Monge / SMAWK} \]