Half-space Farthest point Queries

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1 Problem Definition

Input: Let $P \in \mathbb{R}^d$ be a set of points lying in convex position. Also given are a point $p \in P$ and a halfspace $\hat{h} \in \mathbb{R}^d$.

Output: The point $q \in P$ which is farthest from p in the halfspace, i.e.

$$q = \operatorname*{argmax}_r \left\{ \text{dist}(p, r) \,\middle|\, r \in P \cap \hat{h} \right\}$$

This problem commonly arises when we try to approximate a given set of convex point P by another set of points P' ([1, 2]).

2 Things known

For $d = 2$, we have to compute the farthest point from a query point which lies on the 'left side' of a given ray. [2] provides a data-structure to answer queries in $O\left(\log^2 n\right)$ time with $O\left(n \log n\right)$ preprocessing time and space complexity of the data-structure. [1] provides two improved data-structures:

- First data-structure uses $O\left(n^{1+\epsilon}\right)$ space and preprocessing time; and answers queries in $O\left(2^{1/\epsilon} \log n\right)$ time.
- Second data-structure uses $O\left(n \log^3 n\right)$ space and polynomial preprocessing time; and answers queries in $O\left(\log n\right)$ time.

3 What we Might want to solve

Solving the problem for $d = 3$ should be a good starting point. As far as my knowledge goes, this problem has not yet been solved. Any data-structure with polynomial space complexity and pre-processing time which supports $o(n)$ query time should be an interesting result.

References

[1] Boris Aronov, Prosenjit Bose, Erik D. Demaine, Joachim Gudmundsson, John Iacono, Stefan Langerman, and Michiel Smid. Data structures for halfplane proximity queries and incremental voronoi diagrams. *LATIN* (2006).

[2] Ovidio Daescu, Ningfang Mi, Chan-Su Shin, and Alexander Wolff. Farthest-point queries with geometric and combinatorial constraints. *Computat. Geom. Theory Appl.* (2006).