Exploring tighter bounds for the 2-dimensional spatial search variant of Grover's algorithm*

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1 History of Quantum Computation.

In 1982, Richard Feynman introduced the notion of simulating quantum-mechanical objects by using other quantum systems [F82]. The full power of quantum computers was shown in a 1985 paper by David Deutsch which described a universal quantum computer [D85]. Everyone agreed that the quantum computer was powerful, but nobody could show how to utilize the full capacity of that power. However, in 1994, Peter Shor of AT&T's Bell Laboratories first published a paper [S97] that introduced a quantum algorithm for efficient factorization – a technique that can, in theory, crack current encryption codes in a matter of seconds. After the introduction of Shor's algorithm, quantum computation became an increasingly more interesting and relevant field.

2 Grover's Algorithm and the Spatial Search Variant.

There are few known quantum algorithms, but one of the most influential was introduced by Lov Grover in 1997 [G97]. Grover's algorithm prescribes a method for using a quantum computer to search an unsorted database for a marked item in $O(\sqrt{N})$ quantum steps, a speedup over the classically required $\Omega(N)$ queries. This algorithm has been applied to a wide variety of problems to obtain considerable speedup over the classical analogues.

The spatial search variant of Grover's algorithm adds the constraint that the N database items are stored in N different memory locations, so there will also be a cost involved with moving between spatial locations. This problem is especially interesting in the 2-dimensional case where the N search items are arranged on a grid of size $\sqrt{N}$ x $\sqrt{N}$. In this case, the usual Grover's search algorithm will cost $\Theta(\sqrt{N})$ queries, however a distance of $\Theta(\sqrt{N})$ may be moved between any two queries. Thus, Grover's search algorithm will require $\Omega(N)$ steps – there is no speedup over the classical search solution. However, it may still be possible to find some other quantum algorithm that can perform

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the search as fast as the $\Omega(\sqrt{N})$ lower bound implied by the original version of Grover's algorithm.

Previous work on this problem began with an algorithm by Aaronson and Ambainis for searching the 2-dimensional grid in $O(\sqrt{N} \log^2 N)$ total steps [AA03]. Childs and Goldstone later applied the use of continuous quantum walks to this problem, which yielded some progress for 5 and more dimensions yet required $\Omega(N)$ steps for the 2-dimensional case [CG04]. Most recently, Ambainis, Kempe, and Rivosh showed how to use discrete quantum walks to search the 2-dimensional grid in $O(\sqrt{N} \log N)$ steps [AKR05].

3 Proposal and Conclusion.

I propose as a project to continue searching for tighter bounds to the spatial search variant of Grover's algorithm, specifically to the case of a 2-dimensional space. This could amount to proving tighter lower bounds, or to giving a quantum algorithm that performs better than $O(\sqrt{N} \log N)$ steps.

The quantum spatial search problem has been the topic of much research and puzzlement over the past few years – a fairly long time, considering how young the field of Quantum Complexity Theory is. Making improvements on the 2-dimensional case will prove challenging, since it *has* been significantly researched; however, the recent advances in this problem, as well as advances in quantum lower bounds methods [BBBV97, BBCMW98, A02], can serve as a guide to get us started in (more or less) the right direction.

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