CS573 Data Structures Spring 2006

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With the prevalent use of mobile communication and ad hoc networks, the question of tracking connectivity in this newly dynamic enviornment becomes vital to the preservation of useful communication. Such a network can be viewed as a set of uniquely identifiable mobile hosts where peers communicate without the need for a central authority. Two hosts within each other's transmission radius are allowed to communicate and relay any messages. This problem can be viewed geometrically as a set of moving disks in the plane where each disk represents a host with the same radii, where given two disks we wish to know whether they both fall in a connected component formed by the overlap of the set. We are interested in a kinetic data structure[1] maintaining connectivity of these disks of arbitrary radii.

More precisely, let $c_1, \ldots, c_n$ be functions which dicate the path of motion for the centers of n disks with $r_1, \ldots, r_n$ the radii of those disks. We assume each function c_i is described by some low-degree algebraic curve such that the collision or separation of two disks can be computed in constant time. Thus, the problem is devising a kinetic data structure which maintains connectivity of disks $c_1, \ldots, c_n$ with radii $r_1, \ldots, r_n$.

The determinition of connectivity given disks without motion has been well-studied and several algorithms can be found in standard texts. Similar problems of connectivity with motion have been studied by Guibas, Hershberger, Suri and Zhang including the use of rectangles[2] and unit disks[3]. The problem of extending their unit disks structure to non-unit disks was left as open.

The solution for unit disks originates from the idea of maintaining an overlap graph [4], a graph in which each disk becomes a vertex and an edge exists between vertices u and v if and only if the disks represented by u and v have nonempty intersection. However, instead of merely using this previous work which has worst-case bounds which prevent a compact representation, instead a spanning subgraph of the overlap graph is kept. In identifying the representation, a finite number of regions around any given disk center are used in order to confirm an overlap. Namely, by dividing the right half of a disk of radius 2 into 8 regions and a unit disk into 4 strips, each point can be classified into a connected component based on its region. By using this classification, a multigraph sharing the same connected components as the connectivity graph is determined.

In order to maintain this graph in the face of motion, structures managing all points which fall to the right of a disk within its range of height, entitled its *shadow* are necessary. Via sweeping right to left an initial component structure can be computed, updating only when the sweep line connects with a new disk center. Also maintained is a vertical order of the intersection of disks via their shadow representation, denoted the shadow diagram. Updates to the structure are triggered via the violation of either the x-coordinate ordering, y-coordinate ordering, or an *arc-ordering* (a condition managed via the shadow diagram).

The difficulty in augmenting this strategy seems to lie in an efficient identification of the potential regions in which two disks may trigger an alteration to their status of intersection. However, I have found no other alternative kinetic data structure proposed to solve the connectivity problem for non-unit disks. In fact, the only kinetic connectivity problems which have been studied are those of rectangles and unit disks. Thus, any efficient kinetic data structure would be welcome. It is of use to note that while the structure proposed for unit radii was efficient, it still failed both locality and responsiveness, and thus a solution to non-unit radii likely will not attain these criteria either.

References

[1] Leonidas Guibas. Kinetic data structures. Chapter 23 of *Handbook of Data Structures and Applications* (Dinesh P. Mehta and Sartaj Sahni, eds.), 23-123-18. Chapman and Hall/CRC, 2005

[2] J. Hershberger and S. Suri. Kinetic Connectivity of Rectangles. In *Proc. 15th ACM Symposium on Computational Geometry*, 1999.

[3] Leonidas Guibas, John Hershberger, Subash Suri, and Li Zhang. Kinetic Connectivity for Unit Disks. In *Proc. 16th Annu. ACM Sympos. Comput. Geom.*, pages 331340, 2000.

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