Homework 1 is due in two weeks.
- roughly every other week
  * I don't know
  * I know nobody knows
- Gradescope (details soon)
- teams of 5
- Solutions from you

Straight-line embedding
- Schnyder
- Steinitz/Tutte/Koebe

Duality
- data structures
- dictionary

Euler's Formula

d de Fraysseix, Pach, Pollack
canonical order
To compute these:

Fix a root vertex on boundary (green)
Repeat
identify edge $e_u v$ s.t. $v$ has exactly
2 neighbors in common with $u$
Contract $e_u v$
Delete parallel edges
Recurse
Expand
0(n) time
Every internal vertex has 1 outedge of each color.
No mono cycles

3 trees $\Rightarrow$ “wood”

Regions
Count verts in each regions
$(2, 7, 4)$

$v \Rightarrow w$
$g(w) > g(v)$
$r(w) < r(v)$
$b(w) \leq b(v)$

$\max b \in \{\max g, \max r\}$
Theorem:
Any simple planar \( G \) has a straight embedding with integer vertex coords between 0 and \( n-2 \).

Encoding: 2-bit per edge
\( O(n/\log n) \) space

Greedy routing

Steinitz: 1916

Tutte: 1950s

Spring embedding
3-connected planar
Convex equi-\( k \) embedding

Koebe: 1940s
<table>
<thead>
<tr>
<th>primal G</th>
<th>dual G*</th>
<th>primal G</th>
<th>dual G*</th>
</tr>
</thead>
<tbody>
<tr>
<td>vertex v</td>
<td>face v*</td>
<td>empty loop</td>
<td>spur</td>
</tr>
<tr>
<td>dart d</td>
<td>dart d*</td>
<td>loop</td>
<td>bridge</td>
</tr>
<tr>
<td>edge e</td>
<td>edge e*</td>
<td>cycle</td>
<td>bond</td>
</tr>
<tr>
<td>face f</td>
<td>vertex f*</td>
<td>even subgraph</td>
<td>edge cut</td>
</tr>
<tr>
<td>tail(d)</td>
<td>left(d*)</td>
<td>spanning tree</td>
<td>complement of spanning tree</td>
</tr>
<tr>
<td>head(d)</td>
<td>right(d*)</td>
<td></td>
<td>G \ e</td>
</tr>
<tr>
<td>succ</td>
<td>rev o succ</td>
<td>minor G \ X / Y</td>
<td>G* \ e*</td>
</tr>
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<td></td>
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<td>minor G \ X / Y</td>
<td>G* \ e*</td>
</tr>
</tbody>
</table>

Euler’s Formula \[ V-E+F=2 \] 

[Descartes]

Proof by induction

If \( V=1 \), \( E=0 \), \( F=1 \)  
Pick any edge e
If \( e \) is not a bridge, consider \( G \setminus e \)

\[
\begin{array}{c}
\text{If } e \text{ is not a loop, consider } G \setminus e \\
\end{array}
\]

\[
\begin{array}{c}
\text{Proof 2: [von Staudt 1847]}
T \text{ spanning tree } \rightarrow C^* \text{ cotree}
\end{array}
\]

\[
\begin{array}{c}
E \quad V - 1 \quad F - 1
\end{array}
\]

\[
\begin{array}{c}
\text{Proof 3: [Schnyder]}
\end{array}
\]

Schyneder wood \( R, G, B \)

\[
\begin{array}{c}
E = 3(V-3) \text{ edges} + 3 = 3V - 6
\end{array}
\]

\[
\begin{array}{c}
2E = 3F \quad \Rightarrow \quad F = 2V - 4
\end{array}
\]

\[
\begin{array}{c}
v = (3v - 6) + 2v - 4 = 2
\end{array}
\]