Admin: Traveling Sept 19 and 21 — no class
makeup lectures later
Office hours Fridays (when?) — Doodle
tomorrow 3-4 pm
3304 Sibley

Closed curves $S^1 \to \mathbb{R}^2$ continuous
Input to an algorithm

Three representations
* Polynomials — specified by finite rep of pts $P_0 ... P_n$
  edges $P_0P_1P_2 ... P_n$
  vertices
  Braden 1340s
  Meister 1700

Additional work to find self-crossings
* Walk in plane graph $G$
  curve — closed walk
  geometrically or combinatorially
  $v_1 v_2 v_3 v_4 v_5 v_6 v_7$
- Generic curves/immersions

- All self-intersections are
  - pairwise
  - transverse \( \Rightarrow \) finite

Every curve is arbitrarily close to a generic curve

\[
\delta, \gamma : S^1 \to \mathbb{R}^2 \quad d(\delta, \gamma') = \max_{\theta} ||\delta(\theta) - \gamma'(\theta)||
\]

Represent by image graph, 4-regular or cycle
or Gauss code (maybe sign)

\[\text{abcd efgh b id j fejk h} \text{ tickg}\]

\[\text{abab}\]
Gauss asked: When is a Gauss code planar?

Signed Gauss code

embedding of graph

Planar \iff V - E + F = 2

[Francis Carter 1978]

Dehn's condition

necessary condition

not sufficient

abcdefg fghb cdefjkhcatkk

\[ a^+ b^+ c^- a^- b^- c^+ \]

\[ \text{abc} \rightarrow \text{fgkaq} \rightarrow \text{z} \]

\[ \text{akg} \rightarrow \text{obaq} \rightarrow \text{z} \]
Interleave graph: vertices = symbols

Interleave bipartite $\rightarrow \mathcal{O}(n)$ time