Homotopy = continuous deformation
⇒ equivalence of strings

Today: Shortest homotopic Paths

Geometry of obstacles matters
Environment = polygon with holes

Test homotopy:

Triangulate
Use diagonals to def. crossing seq
Reduce as usual
Sleeve = union of Δs defined by reduced crossing sequence

No holes: Keep one of each symbol that appears odd # times,

Shortest path in Δ-polygon - FUNNEL algorithm
Shrink the fan in $O(1)$ time per deleted vertex

Total # additions $\leq n$

Total time $= O(n)$

Expand fan in $O(1)$ times, adding $m$

Shortest path from $s$ to $t$ in simple triangulated polygon $= O(n)$ time.
Triangulation has n diagonals $+ O(n \log n)$
Input path has k edges $\Delta n$

$O(x) = O(kn)$ — crossing seq
$+ O(x) = O(x)$ — reduce
$+ O(x) = $ Funnel
Covering space

\[ \pi : \tilde{X} \to X \]

universal covering space

\( x \in X \) \( \pi^{-1}(x) \) lifts of \( x \)

Lemma: Given any path \( \alpha : [0,1] \to X \)

there is a unique path \( \tilde{\alpha} : [0,1] \to \tilde{X} \)

s.t. \( \pi \circ \tilde{\alpha} = \alpha \)

Similar for homotopy -
homotopic paths have homotopic lifts

$\alpha : [0,1] \rightarrow X$ is contractible loop
iff $\tilde{\alpha}$ is a loop

Make a copy of each region for each reduced $x$-seq ending in that region.

Name region $(1, ABbA)$

Glue $(r, w)$ to $(r', wx)$ single char along ray $x$