Projects

Solo proposals 2-3 pages 3 weeks

What do you want to do?

Background — Refs/sketch of known results

Proposal — half-baked is okay

Group projects up to 3

- Presentation ~ 20 min
- Write-up ~ 15 pages

Finals week

Planar graphs edges — MSSP

negative lengths SP

Today: minimum cuts
maximum flows

Separators
r-divisions
Monge structure

Monge heap
Coping arguments

Winding arguments

Min cuts

Edges of G have weight (capacity)

We want min-weight (s,t)-cut

Duality — simple cut ↔ simple cycle

Separates separates

s and t

s* and t*

Shortest cycle in an annulus

Separates boundary

Winding #1 around t*

Homotopic to boundary

T = shortest path from s* to t* Intuition: min cycle crosses T once

crossing

not crossing
Define $x$-seq: write $+$ whenever $Y$ crosses $T$ $\rightarrow$

Reduced $x$-seq $= +$ Either reduced or contains $+-+$

IF $Y$ not reduced then not shortest

$Y \cap T$ is a subpath of $T$

Diagram:

Naive: Dijkstra $x \leq k \Rightarrow O(k n \log n) = O(n^2 \log n)$

MSSP $\Rightarrow O(n \log n)$

Divide and conquer (RellF) $\Rightarrow O(n \log n \log k)$

Compute shortest path from $X_{x^+}$ to $X_{x^-}$

cut rect along that path, recurse in both parts

$k = \# \text{vertices in } T$

$n = \# \text{faces} \quad \text{Enter } \Rightarrow \# \text{vertices} = \Theta(n)$

$T(n, k) = O(n \log n) = T(n_1, \frac{k}{2}) + T(n_2, \frac{k}{2}) + O(n \log n \log k)$

$n_1 + n_2 = n$
Italiano et al. [2011] \( \rightarrow O(n \log \log n) \)

\[ O(n \log \log k) \]

\[ O(n \log \log n) \text{ prep.} \]

- compute \( \log^c k \) LRZ shortest paths \( \text{in } O(n) \) time

- \( \text{recursively in each stripe} \)
- \( \text{with each stopping} \)

\[ T(k) = O(n) + \sum_{i=1}^{k} T(n; \frac{k}{f(i)}) \]

Let \( F(k) = \frac{n}{F(k)} \)

\[ T(n, k) = O(nF^*(k)) \]

FRZ - Bellman Ford \hspace{1cm} \text{Exploits Monge structure}

FRZ - Dijkstra

- DDG \( O\left(\frac{n^2}{\log n}\right) \) vertices and \( O(n) \) edges

- \( \text{Dijkstra: } O(E + V \log V) = O(n + \frac{n^2}{\sqrt{\log n}}) \)

BAD
Monge heap

\[ M = \overset{\text{unknown}}{\text{Monge array}} \]

- Reveal a column
- Find min revealed entry
- Kill a row

\[ \text{TS} + C \]

\[ \text{known} \quad \text{not known} \quad \text{const. columns} \]

\[ w(u,v) + \text{dist}(u) \]