homotopy
= continuous deformation

one obstacle
winding number

one finger
loose
both → fast
def signedArea(P):
    area = 0
    n = size(P)
    for i in range(n):
        area += (P[i].x * P[(i+1)%n].y - P[i].y * P[(i+1)%n].x) / 2
    return area
wind(P, 0) = \text{sum of signed angles at 0 subtended by the edges of } P \quad \text{always integer!}

\frac{\pi}{2} \leq \Theta \leq \frac{\pi}{2}

Don't compute it this way!

wind(P, 0) = \# \text{pos crossings} - \# \text{neg crossings}
def windingNumber(P, o):
    wind = 0
    n = size(P)
    for i in range(n):
        p = P[i]
        q = P[(i+1)%n]
        Delta = (p.x - o.x)*(q.y - o.y) - (p.y - o.y)*(q.x - o.x)
        if p.x <= o.x < q.x and Delta > 0:
            wind += 1
        elif q.x <= o.x < p.x and Delta < 0:
            wind -= 1
    return wind

# roots = wind(F(C), 0)
Homotopy = continuous deformation = morph

Homotopy between closed curves in punctured plane $\mathbb{R}^2 \setminus \{0\}$

Continuous function $h: [0,1) \times S^1 \to \mathbb{R}^2 \setminus \{0\}$

$h(0, t)$ is initial curve
$h(1, t)$ is final curve
$h(t, \cdot)$ is intermediate

Even when initial and final curves are nice (polygons), intermediate curves can be nasty.

Two closed curves are homotopic iff there exists a homotopy from one to the other

1. We can model/approx any homotopy by seq. of simple moves

2. Two polygons in $\mathbb{R}^2 \setminus \{0\}$ are homotopic iff

same winding number around 0.
Theorem:
Two polygons in $\mathbb{R}^2 \setminus \{0\}$ are homotopic iff they are connected by a sequence of safe vertex moves.

Proof: $\Leftarrow$ "obvious" (mod definitions)
$\Rightarrow$ hard

Let $h$ be any homotopy from $P_0$ to $P_1$

build grid of $8 \times 8$ squares
image(□) has diameter $\leq \varepsilon$

Move $\Delta$s one at a time above curve
seq of safe vertex moves, each moving a vertex $\leq \varepsilon$

Simplicial approximation theorem
homotopic ⇔ same winding

⇒ wind # only changes when P and o intersect

① remove redundant verts
② subdivide along rays
③ move every vertex to unit circle