Homotopy = continuous deformation.

Polygons in $\mathbb{R}^2 \setminus \Omega$

obstacle pts

crossing sequence $\Rightarrow$ a b c d e f g h

empty $\Rightarrow$ contractible!

$O(k \log k + nk)$ time

k = #obstacles

n = #polygon vertices

$\Omega(nk)$
Suppose $P$ is simple. For each obstacle inside $P$?

$$O(\log n)$$

Preprocess $P$

Is $o$ inside $P$?

$$O(\log n)$$

$$\Rightarrow O((n+k) \log n) \text{ time}$$

Faster contractibility test for polygons with few self-intersections.

Moral: We’re doing topology

So we can modify the geometry.

$$O((n+k+\frac{3}{2}) \log (n+k)) \text{ time}$$

#self-intersections
Given $O$, $P$ but not self-intersections.

Bentley-Ottman
Sweep-line algorithm $O((n+k+s) \log n)$

Maintain seq. of intersections with a vertical line in a balanced BST

Vertex event $\rightarrow O(1)$ indels to both BST and PQ

Intersection event $\rightarrow$

Keep intersections between adjacent edges in BST in a priority queue
Next: $O(\log n)$
vertical + horizontal ranking

vertical ranking: how many obstacles below?
horizontal: to left?

Subdivide edges @ self-intersections
\( S = \text{segments} = \text{polygon edges + obstacles} \)

\[ \sigma_1 \uparrow \sigma_2 \downarrow \sigma_3 \]

is trans. closure of

\[ \sigma \]

partial order

\[ \text{Proof: } \sigma_1 \uparrow \sigma_2 \downarrow \sigma_3 \uparrow \sigma_3 \uparrow \sigma_4 \]

\( \sigma_4 \) left-most right endpoint
\[ v_{\text{rank}}(\sigma) = \begin{cases} 2 \cdot \# \text{obst} \in \sigma \cap \mathbb{Z} \times \mathbb{Z} & \text{if } \sigma \text{ is obst} \\ 2 \cdot \# \{ \text{obst} \in \sigma \cap \mathbb{Z} \times \mathbb{Z} : \text{odd} \} & \text{if } \sigma \text{ is edge} \end{cases} \]

\[ h_{\text{rank}}(p) = \begin{cases} 2 \cdot \# \text{obst to left} + 1 & \text{if } p \text{ obst} \\ 2 \cdot \# \text{obst to left} & \text{if } p \text{ vertex} \end{cases} \]

**Rectification**

- replace obstacle \( o \) with \((h_{\text{rank}}(l_0), v_{\text{rank}}(l_0))\)
- replace edge \( pq \) with \((h_{\text{rank}}(p), v_{\text{rank}}(pa))\)
  \((h_{\text{rank}}(q), v_{\text{rank}}(pq))\)
- replace vertex \( q \) with \((h_{\text{rank}}(q), v_{\text{rank}}(pq))\)
  \((h_{\text{rank}}(q), v_{\text{rank}}(q \rightarrow))\)
Remove all zero-length edges.

Every bracket slide either freezes an edge or deletes $\geq 2$ vertices.

$\Rightarrow O(n)$ slides $\Rightarrow$ halt.
Details

Prove if $P$ is contractible, bracket slides.

**Time analysis**

<table>
<thead>
<tr>
<th>Operation</th>
<th>Time Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trap decompression</td>
<td>$O((n+k+s) \log n)$</td>
</tr>
<tr>
<td>Rectification</td>
<td>$O(n+k+s)$</td>
</tr>
<tr>
<td>Each bracket slide</td>
<td>$O(\log k)$</td>
</tr>
</tbody>
</table>

Stop after $O(n+k+s)$ slides

$\Rightarrow O((n+k+s) \log(n+k))$ time