Paper chase due Monday-ishly
Project proposals - Halloween-ishly

Curve homotopy and curve invariants

Steinitz (1916): A graph is the skeleton of a convex polyhedron in $\mathbb{R}^3$ iff planar and 3-connected.

Any planar curve with $n$ vertices can be simplified with $O(n^2)$ moves.

Proof: Repeat until simple:
If empty loop/monogon $1 \rightarrow 0$
else find loop
Find minimal loop contains no loops
def \( \lambda \) = 0

\( \text{def} \) \( (\lambda) \) = 0

Monotone: Steinitz is best known

Proof: always known

\( [CH2] \)

\( B \sum_{i=1}^{n} \mathbf{a}_{i} \)

1 - 0 - 2 - 0 - 3 - 3

3 - 3 - 3 moves. 2 - 0 - 2 - 0 - 3 - 3

0 (\( \mathbf{a}_{i} \)) moves. 2 - 0 - 2 - 0 - 3 - 3

\( \text{curve invariant} \)

\( \text{Find eigenvalues} \)

\( \text{minimal lens} \)

\( \text{minimal} \)
Rotation number $= \text{tangent winding } \#$

$\text{rot} = 0$

$\text{rot}(\gamma) := \text{wind}(\gamma', 0)$

$\text{rot} = \mathbb{Z} \text{ extangles } \in \mathbb{Z}$

$\text{wind}(\gamma', 0) = \# \gamma' - \# \gamma$

$\text{rot}(\gamma) = 2 \text{wind}(\gamma, \gamma(0))$

$+\sum \text{sgn}(x)$

$-\frac{1}{2}$

$-1$

$0$

$\frac{1}{2}$

$0$

$\frac{1}{2}$

$1$

$0$

$1$

$2$

$\frac{1}{2}$

$0$

$1$

$2$

$\gamma$

$\gamma(0)$

$\text{rot}(\gamma) = \geq \text{rot}(\gamma_i)$

$\# \gamma_i - \# \gamma$

$\text{wind}(\gamma, \gamma(0)) = \frac{1}{\varepsilon}$

$\text{writhe } (\gamma) = 1$

$\text{rot} = 2$
Proof: Base point independent

\[
\text{defect} = (\text{wind}(\gamma, \gamma b)^2 - \frac{1}{4}) + \sum_{x \neq y} \text{sgn}(x) \cdot \text{sgn}(y)
\]

\[
\text{interlaced}
\]

Lemma: \( \Box \Rightarrow \Box \) defect unchanged

\( \Delta \text{defect} \text{ either } 0 \text{ or } -2 \)

\( \Delta \text{defect} = \pm 2 \)

Proof: case analysis
Defect(0) = 0 \quad \# \text{moves}(\gamma) \geq \frac{1}{2} \text{defect}(0)

Flat Torsion knot / Spirograph curve

\[ T(p,q)(\theta) = \left( \cos(q\theta) + 2, \cos(p\theta), (\cos(q\theta)+2) \sin(p\theta) \right) \]

defect(T(p,p+1)) = 2 \binom{p+1}{3}

\[ \uparrow \quad n = \Theta(p^2) \quad \uparrow \quad \Theta(p^3) = \Theta(n^{3/2}) \]