Proposals due Friday 13th → Gradescope

Next Tue — election day — no live lecture
Next Thu — present proposals

Plane. Map \( \Sigma = (V, E, F) \)

Surface

\[ V - E + F = 2 \]

Translation system \((D, \text{rev}, \text{succ})\)

\[ \text{succ}(d) = \text{next dart in cw order around head} \]

\[ 2\text{-manifold} / \text{surface} = \mathbb{C} \]

Space * locally homeomorphic to \( \mathbb{R}^2 \)

For all \( x \in S \), there is open set \( x \in U \subseteq S \)

\[ U \cong \mathbb{R}^2 \]

2-manifold / surface = \( \mathbb{C} \)

Polygonal schema \((\preceq, \text{dual rotation system})\)

\[ \text{succ}^* = \text{rev} \circ \text{succ} \]

- For each face \( F \), corresponding disk
- For any face of degree \( d \), divide ccw
  - boundary of disk into all directed paths
  - paths \( \cong \) darts
Glue identify paths $d$ and $\text{rev}(d)$ for all $d(t) \sim \text{rev}(d)(t-t)$

$$S = F/\sim$$

**Non-orientable surfaces**

Möbius band

Surface $S$ is orientable iff it contains no Möbius bands

**Orientable = sphere + handles**

genus = "# handles"

# cycles that can be deleted/sliced/cut without disconnecting surface
Projective Plane

V = 1
E = 1
F = 1
v - E + F = 1
genus 1

Klein Bottle

genus 2

Band Decomposition

vertices \rightarrow \text{blade/flag of } \Sigma

to change v \rightarrow \text{sides}

to change e \rightarrow \text{corner}

to change f \rightarrow \text{darts}

Faces \rightarrow \text{edges}

Reflection system

(\Phi, a, b, c)

- \Phi - finite set of flags or blades
- \alpha: \Phi \rightarrow \Phi \text{ involution} \rightarrow \text{apex}
- b: \Phi \rightarrow \Phi \text{ involution} \rightarrow \text{border}
- c: \Phi \rightarrow \Phi \text{ involution} \rightarrow \text{chamber}
- \alpha \circ c = c \circ \alpha = \text{id}

vertices of \Sigma = \text{orbits of } \langle b, c \rangle

edges \langle \alpha \rangle

faces \langle a, b \rangle
Duality

\[ Z = (D, \text{rev, succ}) \rightarrow E^*(D, \text{rev, rev(succ)}) \]

\[ \Xi = (\Phi, a, b, c) \rightarrow \Xi^* = (\Phi, c, b, a) \]

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**Surface Classification Theorem**

These are the only surfaces:

- Sphere orientable with any genus \( g \geq 0 \)
- Non-orientable with any genus \( g \geq 0 \)

\[ V - E + F = 2 \]

\[ V - E + F = 2 - 2g \]

Deletion + Contraction

Tree + Cotree Decompositions

Deletion:

\[ (\Phi, a, b, c) \]

Contrast:

\[ (\Phi) e, z, b/e, c \]

\[ (b/e)(\phi) = \begin{cases} b\phi(c(b\phi)) & \text{if } b\phi \text{ is even} \\ b\phi c(b\phi) & \text{if } b\phi \text{ is odd} \\ b\phi & \text{else} \end{cases} \]