

Bibliography

- [1] Pankaj K. Agarwal. Partitioning arrangements of lines: II. Applications. *Discrete Comput. Geom.*, 5:533–573, 1990.
- [2] Pankaj K. Agarwal, N. Alon, B. Aronov, and S. Suri. Can visibility graphs be represented compactly? In *Proc. 9th Annu. ACM Sympos. Comput. Geom.*, pages 338–347, 1993.
- [3] Pankaj K. Agarwal, D. Eppstein, and J. Matoušek. Dynamic half-space reporting, geometric optimization, and minimum spanning trees. In *Proc. 33rd Annu. IEEE Sympos. Found. Comput. Sci.*, pages 80–89, 1992.
- [4] Pankaj K. Agarwal and J. Matoušek. Ray shooting and parametric search. *SIAM J. Comput.*, 22(4):794–806, 1993.
- [5] A. Aggarwal, M. Hansen, and T. Leighton. Solving query-retrieval problems by compacting Voronoi diagrams. In *Proc. 22nd Annu. ACM Sympos. Theory Comput.*, pages 331–340, 1990.
- [6] Nancy M. Amato and Edgar A. Ramos. On computing Voronoi diagrams by divide-prune-and-conquer. In *Proc. 12th Annu. ACM Sympos. Comput. Geom.*, pages 166–175, 1996.
- [7] Nina Amenta and Günter Ziegler. Deformed products and maximal shadows of polytopes. Report 502-1996, Technische Universität Berlin, May 1996. Available electronically at <ftp://ftp.math.tu-berlin.de/pub/Preprints/combi/Report-502-1996.ps.Z>.
- [8] Nina Amenta and Günter Ziegler. Shadows and slices of polytopes. In *Proc. 12th Annu. ACM Sympos. Comput. Geom.*, pages 10–19, 1996.

- [9] Arne Anderson, Torben Hagerup, Stefan Nilsson, and Rajeev Raman. Sorting in linear time? In *Proc. 27th Annu. ACM Sympos. Theory Comput.*, pages 427–436, 1995.
- [10] D. Avis and K. Fukuda. A pivoting algorithm for convex hulls and vertex enumeration of arrangements and polyhedra. *Discrete Comput. Geom.*, 8:295–313, 1992.
- [11] David Avis and David Bremner. How good are convex hull algorithms? In *Proc. 11th Annu. ACM Sympos. Comput. Geom.*, pages 20–28, 1995.
- [12] David Avis, David Bremner, and Raimund Seidel. How good are convex hull algorithms? *Comput. Geom. Theory Appl.*, 1996. To appear. Full version of [11]. Available electronically at <ftp://mutt.cs.mcgill.ca/pub/doc/hgch.ps.gz>.
- [13] Saugata Basu. On bounding the Betti numbers and computing the Euler characteristic of semi-algebraic sets. In *Proc. 28th Annu. ACM Sympos. Theory Comput.*, pages 408–417, 1996.
- [14] Saugata Basu, Richard Pollack, and Marie-Françoise Roy. On the number of cells defined by a family of polynomials on a variety. *Mathematika*. To appear. Available electronically at <http://www.math.nyu.edu/faculty/pollack/finalvariety.ps>.
- [15] Walter Baur and Volker Strassen. The complexity of partial derivatives. *Theor. Comput. Sci.*, 22:317–330, 1983.
- [16] M. Ben-Or. Lower bounds for algebraic computation trees. In *Proc. 15th Annu. ACM Sympos. Theory Comput.*, pages 80–86, 1983.
- [17] Samuel W. Bent and John W. John. Finding the median requires $2n$ comparisons. In *Proc. 17th ACM Sympos. Theory Comput.*, pages 213–216, 1985.
- [18] A. Björner, M. Las Vergnas, N. White, B. Sturmfels, and G. Ziegler. *Oriented Matroids*. Cambridge University Press, Cambridge, 1993.
- [19] Anders Björner, László Lovász, and Andrew C. C. Yao. Linear decision trees: Volume estimates and topological bounds. In *Proc. 24th Annu. ACM Sympos. Theory Comput.*, pages 170–177, 1992.

- [20] S. Bloch, J. Buss, and J. Goldsmith. How hard are n^2 -hard problems? *SIGACT News*, 25(2):83–85, 1994.
- [21] Manuel Blum, Robert W. Floyd, Vaughan Pratt, Ronald R. Rivest, and Robert E. Tarjan. Time bounds for selection. *J. Comput. System Sci.*, 7(4):448–461, 1973.
- [22] J. Bochnak, M. Coste, and M.-F. Roy. *Géométrie algébrique réelle*, volume 12 of *Ergebnisse der Mathematik und ihrer Grenzgebiete 3*. Springer-Verlag, 1987.
- [23] H. Brönnimann, B. Chazelle, and J. Pach. How hard is halfspace range searching? *Discrete Comput. Geom.*, 10:143–155, 1993.
- [24] Stefan A. Burr, Branko Grünbaum, and N. J. A. Sloane. The orchard problem. *Geom. Dedicata*, 2:397–424, 1974.
- [25] J. Canny. Some algebraic and geometric configurations in PSPACE. In *Proc. 20th Annu. ACM Sympos. Theory Comput.*, pages 460–467, 1988.
- [26] J. Canny. Computing roadmaps in general semialgebraic sets. *Comput. J.*, 36:409–418, 1994.
- [27] Timothy M. Chan. Fixed-dimensional linear programming queries made easy. In *Proc. 12th Annu. ACM Sympos. Comput. Geom.*, pages 284–290, 1996.
- [28] Timothy M. Chan, Jack Snoeyink, and Chee-Keng Yap. Output-sensitive construction of polytopes in four dimensions and clipped Voronoi diagrams in three. In *Proc. 6th ACM-SIAM Sympos. Discrete Algorithms (SODA '95)*, pages 282–291, 1995.
- [29] Timothy M. Y. Chan. Output-sensitive results on convex hulls, extreme points, and related problems. In *Proc. 11th Annu. ACM Sympos. Comput. Geom.*, pages 10–19, 1995.
- [30] D. R. Chand and S. S. Kapur. An algorithm for convex polytopes. *J. ACM*, 17:78–86, 1970.
- [31] R. Chandrasekaran, Santosh N. Kabadi, and Katta G. Murty. Some NP-complete problems in linear programming. *Oper. Res. Lett.*, 1:101–104, 1982.
- [32] B. Chazelle. Reporting and counting segment intersections. *J. Comput. Syst. Sci.*, 32:156–182, 1986.

- [33] B. Chazelle. Lower bounds on the complexity of polytope range searching. *J. Amer. Math. Soc.*, 2:637–666, 1989.
- [34] B. Chazelle. Lower bounds for orthogonal range searching, I: the reporting case. *J. ACM*, 37:200–212, 1990.
- [35] B. Chazelle. Cutting hyperplanes for divide-and-conquer. *Discrete Comput. Geom.*, 9(2):145–158, 1993.
- [36] B. Chazelle. An optimal convex hull algorithm in any fixed dimension. *Discrete Comput. Geom.*, 10:377–409, 1993.
- [37] B. Chazelle. A spectral approach to lower bounds. In *Proc. 35th Annu. IEEE Sympos. Found. Comput. Sci.*, pages 674–682, 1994.
- [38] B. Chazelle. Lower bounds for off-line range searching. In *Proc. 27th Annu. ACM Sympos. Theory Comput.*, pages 733–740, 1995.
- [39] B. Chazelle, L. J. Guibas, and D. T. Lee. The power of geometric duality. *BIT*, 25:76–90, 1985.
- [40] B. Chazelle and J. Matoušek. Derandomizing an output-sensitive convex hull algorithm in three dimensions. Technical report, Dept. Comput. Sci., Princeton Univ., 1992.
- [41] B. Chazelle and B. Rosenberg. Computing partial sums in multidimensional arrays. In *Proc. 5th Annu. ACM Sympos. Comput. Geom.*, pages 131–139, 1989.
- [42] B. Chazelle and B. Rosenberg. The complexity of computing partial sums off-line. *Internat. J. Comput. Geom. Appl.*, 1(1):33–45, 1991.
- [43] B. Chazelle, M. Sharir, and E. Welzl. Quasi-optimal upper bounds for simplex range searching and new zone theorems. *Algorithmica*, 8:407–429, 1992.
- [44] Bernard Chazelle and Burton Rosenberg. Simplex range reporting on a pointer machine. *Comput. Geom. Theory Appl.*, 5:237–247, 1996.
- [45] F. R. K. Chung, P. Erdős, and J. Spencer. On the decomposition of graphs into complete bipartite subgraphs. In Paul Erdős, editor, *Studies in pure mathematics*, pages 95–101. Birkhäuser, 1983.

- [46] K. Clarkson, H. Edelsbrunner, L. Guibas, M. Sharir, and E. Welzl. Combinatorial complexity bounds for arrangements of curves and spheres. *Discrete Comput. Geom.*, 5:99–160, 1990.
- [47] K. L. Clarkson. New applications of random sampling in computational geometry. *Discrete Comput. Geom.*, 2:195–222, 1987.
- [48] K. L. Clarkson. A Las Vegas algorithm for linear programming when the dimension is small. In *Proc. 29th Annu. IEEE Sympos. Found. Comput. Sci.*, pages 452–456, 1988.
- [49] K. L. Clarkson. More output-sensitive geometric algorithms. In *Proc. 35th Annu. IEEE Sympos. Found. Comput. Sci.*, pages 695–702, 1994.
- [50] K. L. Clarkson and P. W. Shor. Applications of random sampling in computational geometry, II. *Discrete Comput. Geom.*, 4:387–421, 1989.
- [51] R. Cole, M. Sharir, and C. K. Yap. On k -hulls and related problems. *SIAM J. Comput.*, 16:61–77, 1987.
- [52] T. H. Cormen, C. E. Leiserson, and R. L. Rivest. *Introduction to Algorithms*. The MIT Press, Cambridge, Mass., 1990.
- [53] M. de Berg, M. Overmars, and O. Schwarzkopf. Computing and verifying depth orders. In *Proc. 8th Annu. ACM Sympos. Comput. Geom.*, pages 138–145, 1992.
- [54] M. de Berg and O. Schwarzkopf. Cuttings and applications. Report RUU-CS-92-26, Dept. Comput. Sci., Utrecht Univ., Utrecht, Netherlands, August 1992.
- [55] M. Dietzfelbinger. Lower bounds for sorting of sums. *Theoret. Comput. Sci.*, 66:137–155, 1989.
- [56] Martin Dietzfelbinger and Wolfgang Maass. Lower bound arguments with “inaccessible” numbers. *J. Comput. Syst. Sci.*, 36:313–335, 1988.
- [57] D. P. Dobkin and D. G. Kirkpatrick. Determining the separation of preprocessed polyhedra – a unified approach. In *Proc. 17th Internat. Colloq. Automata Lang. Program.*, volume 443 of *Lecture Notes in Computer Science*, pages 400–413. Springer-Verlag, 1990.

- [58] D. P. Dobkin and R. J. Lipton. On the complexity of computations under varying sets of primitives. *J. Comput. Syst. Sci.*, 18:86–91, 1979.
- [59] David Dobkin and Richard J. Lipton. A lower bound of $\frac{1}{2}n^2$ on linear search programs for the knapsack problem. *J. Comput. Syst. Sci.*, 16(3):413–417, 1978.
- [60] Charles Lutwidge Dodgson. *St. James Gazette*, August 1, 1883, pages 5–6.
- [61] M. E. Dyer. The complexity of vertex enumeration methods. *Math. Oper. Res.*, 8:381–402, 1983.
- [62] H. Edelsbrunner. *Algorithms in Combinatorial Geometry*, volume 10 of *EATCS Monographs on Theoretical Computer Science*. Springer-Verlag, Heidelberg, West Germany, 1987.
- [63] H. Edelsbrunner, L. Guibas, J. Hershberger, R. Seidel, M. Sharir, J. Snoeyink, and E. Welzl. Implicitly representing arrangements of lines or segments. *Discrete Comput. Geom.*, 4:433–466, 1989.
- [64] H. Edelsbrunner, L. Guibas, and M. Sharir. The complexity of many cells in arrangements of planes and related problems. *Discrete Comput. Geom.*, 5:197–216, 1990.
- [65] H. Edelsbrunner and L. J. Guibas. Topologically sweeping an arrangement. *J. Comput. Syst. Sci.*, 38:165–194, 1989. Corrigendum in 42 (1991), 249–251.
- [66] H. Edelsbrunner, L. J. Guibas, and M. Sharir. The complexity and construction of many faces in arrangements of lines and of segments. *Discrete Comput. Geom.*, 5:161–196, 1990.
- [67] H. Edelsbrunner and E. P. Mücke. Simulation of simplicity: a technique to cope with degenerate cases in geometric algorithms. *ACM Trans. Graph.*, 9:66–104, 1990.
- [68] H. Edelsbrunner, J. O’Rourke, and R. Seidel. Constructing arrangements of lines and hyperplanes with applications. *SIAM J. Comput.*, 15:341–363, 1986.
- [69] H. Edelsbrunner, R. Seidel, and M. Sharir. On the zone theorem for hyperplane arrangements. *SIAM J. Comput.*, 22(2):418–429, 1993.

- [70] H. Edelsbrunner and M. Sharir. A hyperplane incidence problem with applications to counting distances. In P. Gritzman and B. Sturmfels, editors, *Applied Geometry and Discrete Mathematics: The Victor Klee Festschrift*, volume 4 of *DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, pages 253–263. AMS Press, 1991.
- [71] I. Emiris and J. Canny. A general approach to removing degeneracies. In *Proc. 32nd Annu. IEEE Sympos. Found. Comput. Sci.*, pages 405–413, 1991.
- [72] P. Erdős and G. Purdy. Two combinatorial problems in the plane. *Discrete Comput. Geom.*, 13(3–4):441–443, 1995.
- [73] J. Erickson and R. Seidel. Better lower bounds on detecting affine and spherical degeneracies. *Discrete Comput. Geom.*, 13:41–57, 1995.
- [74] Jeff Erickson. Lower bounds for linear satisfiability problems. In *Proc. 6th ACM-SIAM Sympos. Discrete Algorithms (SODA '95)*, pages 388–395, 1995.
- [75] Jeff Erickson. On the relative complexities of some geometric problems. In *Proc. 7th Canad. Conf. Comput. Geom.*, pages 85–90, 1995.
- [76] Jeff Erickson. New lower bounds for convex hull problems in odd dimensions. In *Proc. 12th Annu. ACM Sympos. Comput. Geom.*, pages 1–9, 1996.
- [77] Jeff Erickson. New lower bounds for halfspace emptiness. In *Proc. 37th Annu. IEEE Sympos. Found. Comput. Sci.*, page to appear, 1996.
- [78] Jeff Erickson. New lower bounds for Hopcroft’s problem. *Disc. Comput. Geom.*, 1996. Special issue of papers from the 11th ACM Sympos. Comput. Geom., to appear.
- [79] Stefan Felsner. On the number of arrangements of pseudolines. In *Proc. 12th Annu. ACM Sympos. Comput. Geom.*, pages 30–37, 1996.
- [80] M. L. Fredman. How good is the information theory bound in sorting? *Theoret. Comput. Sci.*, 1:355–361, 1976.
- [81] M. L. Fredman. A lower bound on the complexity of orthogonal range queries. *J. ACM*, 28:696–705, 1981.

- [82] M. L. Fredman. Lower bounds on the complexity of some optimal data structures. *SIAM J. Comput.*, 10:1–10, 1981.
- [83] Michael Fredman and Dan E. Willard. Surpassing the information-theoretic bound with fusion trees. *J. Comput. Syst. Sci.*, 47(3):424–436, 1993. The extended abstract (STOC 1990) had a much more colorful title.
- [84] Z. Füredi and I. Palásti. Arrangements of lines with a large number of triangles. *Proc. Amer. Math. Soc.*, 92(4):561–566, 1984.
- [85] A. Gajentaan and M. H. Overmars. n^2 -hard problems in computational geometry. Report RUU-CS-93-15, Dept. Comput. Sci., Utrecht Univ., Utrecht, Netherlands, April 1993.
- [86] A. Gajentaan and M. H. Overmars. On a class of $O(n^2)$ problems in computational geometry. *Comput. Geom. Theory Appl.*, 5:165–185, 1995.
- [87] David Gale. Neighborly and cyclic polytopes. In V. Klee, editor, *Convexity*, volume VII of *Proc. Symposia in Pure Mathematics*, pages 225–232. Amer. Math. Soc., 1963.
- [88] J. E. Goodman and R. Pollack. Multidimensional sorting. *SIAM J. Comput.*, 12:484–507, 1983.
- [89] J. E. Goodman and R. Pollack. Allowable sequences and order types in discrete and computational geometry. In J. Pach, editor, *New Trends in Discrete and Computational Geometry*, volume 10 of *Algorithms and Combinatorics*, pages 103–134. Springer-Verlag, 1993.
- [90] J. E. Goodman, R. Pollack, and B. Sturmfels. The intrinsic spread of a configuration in \mathbf{R}^d . *J. Amer. Math. Soc.*, 3:639–651, 1990.
- [91] R. L. Graham. An efficient algorithm for determining the convex hull of a finite planar set. *Inform. Process. Lett.*, 1:132–133, 1972.
- [92] Ronald R. Graham, Donald E. Knuth, and Oren Patashnik. *Concrete Mathematics: A Foundation for Computer Science*. Addison-Wesley, Reading, Massachusetts, 1989.

- [93] Dima Grigoriev, Marek Karpinski, Friedhelm Meyer auf der Heide, and Roman Smolensky. A lower bound for randomized algebraic decision trees. In *Proc. 28th Annu. ACM Sympos. Theory Comput.*, pages 612–619, 1996.
- [94] Dima Grigoriev, Marek Karpinski, and Nicolai Vorobjov. Improved lower bound on testing membership to a polyhedron by algebraic decision trees. In *Proc. 36th Annu. IEEE Sympos. Found. Comput. Sci.*, pages 258–265, 1995.
- [95] B. Grünbaum. *Convex Polytopes*. Wiley, New York, NY, 1967. Revised edition, V. Klee and P. Kleinschmidt, editors, *Graduate Texts in Mathematics*, Springer-Verlag, in preparation.
- [96] Branko Grünbaum. *Arrangements and Spreads*. Number 10 in Regional Conf. Ser. Math. Amer. Math. Soc., Providence, RI, 1972.
- [97] H. Günzel. The universal partition theorem for oriented matroids. *Discrete Comput. Geom.* To appear.
- [98] G. Hardy and E. Wright. *The Theory of Numbers*. Oxford University Press, London, England, 4th edition, 1965.
- [99] Laurent Hyafil. Bounds for selection. *SIAM J. Comput.*, 5(1):109–114, 1976.
- [100] Carl Gustav Jakob Jacobi. De functionibus alternantibus earumque divisione per productum e differentiis elementorum conflatum. *J. Reine Angew. Mathematik*, 22:360–371, 1841. Reprinted in *Gesammelten Werke III*, G. Reimer, Berlin, 1884.
- [101] M. J. Katz and M. Sharir. An expander-based approach to geometric optimization. In *Proc. 9th Annu. ACM Sympos. Comput. Geom.*, pages 198–207, 1993.
- [102] S. S. Kislicyn. On the selection of the k th element of an ordered set by pairwise comparisons. *Sibirskii Matematičeskii Žurnal*, 5:557–564, 1964. In Russian.
- [103] D. E. Knuth. *Fundamental Algorithms*, volume 1 of *The Art of Computer Programming*. Addison-Wesley, Reading, MA, 2nd edition, 1973.
- [104] D. E. Knuth. *Sorting and Searching*, volume 3 of *The Art of Computer Programming*. Addison-Wesley, Reading, MA, 1973.

- [105] Donald E. Knuth. *Axioms and Hulls*, volume 606 of *Lecture Notes in Computer Science*. Springer-Verlag, Heidelberg, Germany, 1992.
- [106] László Lovász. Communication complexity: A survey. In Bernhard Korte, László Lovász, Hans Jürgen Prömel, and Alexander Schrijver, editors, *Paths, Flows, and VLSI Layout*, volume 9 of *Algorithms and Combinatorics*, pages 235–265. Springer-Verlag, 1990.
- [107] J. Matoušek. Reporting points in halfspaces. *Comput. Geom. Theory Appl.*, 2(3):169–186, 1992.
- [108] J. Matoušek and O. Schwarzkopf. On ray shooting in convex polytopes. *Discrete Comput. Geom.*, 10(2):215–232, 1993.
- [109] J. Matoušek, M. Sharir, and E. Welzl. A subexponential bound for linear programming. In *Proc. 8th Annu. ACM Sympos. Comput. Geom.*, pages 1–8, 1992.
- [110] J. Matoušek. Linear optimization queries. *J. Algorithms*, 14:432–448, 1993.
- [111] J. Matoušek. Range searching with efficient hierarchical cuttings. *Discrete Comput. Geom.*, 10(2):157–182, 1993.
- [112] J. Matoušek. Geometric range searching. *ACM Comput. Surv.*, 26:421–461, 1994.
- [113] N. Megiddo. Linear programming in linear time when the dimension is fixed. *J. ACM*, 31:114–127, 1984.
- [114] F. Meyer auf der Heide. A polynomial time linear search algorithm for the n -dimensional knapsack problem. *J. ACM*, 31:668–676, 1984.
- [115] J. Milnor. On the betti numbers of real algebraic varieties. *Proc. Amer. Math. Soc.*, 15:275–280, 1964.
- [116] Nicoali E. Mnëv. The universality theorems on the classification problem of configuration varieties and convex polytopes varieties. In O. Y. Viro, editor, *Topology and Geometry—Rohlin Seminar*, volume 1346 of *Lecture Notes in Mathematics*, pages 527–544. Springer-Verlag, 1988.
- [117] Jaroslav Morávek. On the complexity of discrete programming problems. *Apl. Mat.*, 14:442–474, 1969.

- [118] Jaroslav Morávek. A localization problem in geometry and complexity of discrete programming. *Kybernetika*, 8:498–516, 1972.
- [119] János Pach. Personal communication.
- [120] I. G. Petrovskiĭ and O. A. Oleĭnik. On the topology of real algebraic surfaces. *Izvestia Akad. Nauk SSSR. Ser. Mat.*, 13:389–402, 1949. In Russian.
- [121] I. G. Petrovskiĭ and O. A. Oleĭnik. *On the topology of real algebraic surfaces*, volume 70 of *Amer. Math. Soc. Translation*. 1952. 20 pages.
- [122] Vaughan R. Pratt and Foong Frances Yao. On lower bounds for computing the i -th largest element. In *Proc. 14th Annu. IEEE Sympos. Switching and Automata Theory*, pages 70–81, 1973.
- [123] F. P. Preparata and S. J. Hong. Convex hulls of finite sets of points in two and three dimensions. *Commun. ACM*, 20:87–93, 1977.
- [124] A. Prestel. *Lectures on Formally Real Fields*, volume 1093 of *Lecture Notes in Mathematics*. Springer-Verlag, 1984.
- [125] M. O. Rabin. Proving simultaneous positivity of linear forms. *J. Comput. Syst. Sci.*, 6:639–650, 1972.
- [126] E. M. Reingold. On the optimality of some set algorithms. *J. ACM*, 19:649–659, 1972.
- [127] J. Richter-Gebert and G. M. Ziegler. Realization spaces of 4-polytopes are universal. *Bull. Amer. Math. Soc.*, 32(4), October 1995.
- [128] Jürgen Richter-Gebert. Mněv’s universality theorem revisited. Séminaire Lotaringien de Combinatoire, 1995. 15 pages. Available electronically at <http://winnie.math.tu-berlin.de/~richter/partition.ps.Z>.
- [129] Jürgen Richter-Gebert. *Realization Spaces of 4-Polytopes are Universal*. Habilitationsschrift, Technische Universität Berlin, May 1995. Lecture Notes in Mathematics, Springer-Verlag, in preparation. Available electronically at <http://winnie.math.tu-berlin.de/~richter/universality.ps.Z>.

- [130] G. Rote. Degenerate convex hulls in high dimensions without extra storage. In *Proc. 8th Annu. ACM Sympos. Comput. Geom.*, pages 26–32, 1992.
- [131] Issai Schur. *Über eine Klasse von Matrizen, die sich einer gegebenen Matrix zuordnen lassen*. Thesis, Berlin, 1901. Reprinted in *Gesammelte Abhandlungen*, Springer, 1973.
- [132] R. Seidel. A convex hull algorithm optimal for point sets in even dimensions. M.Sc. thesis, Dept. Comput. Sci., Univ. British Columbia, Vancouver, BC, 1981. Report 81/14.
- [133] R. Seidel. A method for proving lower bounds for certain geometric problems. In G. T. Toussaint, editor, *Computational Geometry*, pages 319–334. North-Holland, Amsterdam, Netherlands, 1985.
- [134] R. Seidel. Constructing higher-dimensional convex hulls at logarithmic cost per face. In *Proc. 18th Annu. ACM Sympos. Theory Comput.*, pages 404–413, 1986.
- [135] R. Seidel. A simple and fast incremental randomized algorithm for computing trapezoidal decompositions and for triangulating polygons. *Comput. Geom. Theory Appl.*, 1:51–64, 1991.
- [136] R. Seidel. Small-dimensional linear programming and convex hulls made easy. *Discrete Comput. Geom.*, 6:423–434, 1991.
- [137] P. W. Shor. Stretchability of pseudolines is NP-hard. In *Applied Geometry and Discrete Mathematics: The Victor Klee Festschrift*, volume 4 of *DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, pages 531–554. AMS Press, 1991.
- [138] J. M. Steele and A. C. Yao. Lower bounds for algebraic decision trees. *J. Algorithms*, 3:1–8, 1982.
- [139] William Steiger and Ileana Streinu. A pseudo-algorithmic separation of lines from pseudo-lines. *Inform. Process. Lett.*, 53(5):295–299, 1995.
- [140] J. Stolfi. *Oriented Projective Geometry: A Framework for Geometric Computations*. Academic Press, New York, NY, 1991.

- [141] Volker Strassen. Die Berechnungskomplexität von elementarsymmetrischen Funktionen und von Interpolationskoeffizienten. *Numer. Math.*, 20:238–251, 1973.
- [142] G. F. Swart. Finding the convex hull facet by facet. *J. Algorithms*, 6:17–48, 1985.
- [143] E. Szemerédi and W. T. Trotter, Jr. Extremal problems in discrete geometry. *Combinatorica*, 3:381–392, 1983.
- [144] R. E. Tarjan. A class of algorithms which require nonlinear time to maintain disjoint sets. *J. Comput. Syst. Sci.*, 18:110–127, 1979.
- [145] T. G. Tarján. Complexity of lattice-configurations. *Studia Sci. Math. Hungar.*, 10:203–211, 1975.
- [146] A. Tarski. *A Decision Method for Elementary Algebra and Geometry*. University of California Press, Berkeley, CA, 1951. Prepared for publication by J. C. C. McKinsey.
- [147] R. Thom. Sur l’homologie des variétés algébriques réelles. In S. S. Cairns, editor, *Differential and Combinatorial Topology*, pages 225–265. Princeton Univ. Press, 1965.
- [148] Hing F. Ting and Andrew C. Yao. A randomized algorithm for finding maximum with $O((\log n)^2)$ polynomial tests. *Inform. Process. Lett.*, 49(1):39–43, 1994.
- [149] Zsolt Tuza. Covering of graphs by complete bipartite subgraphs; complexity of 0-1 matrices. *Combinatorica*, 4:111–116, 1984.
- [150] P. M. Vaidya. Space-time tradeoffs for orthogonal range queries. *SIAM J. Comput.*, 18:748–758, 1989.
- [151] J. van Leeuwen. Problem P20. *Bull. EATCS*, 19:150, 1983.
- [152] H. E. Warren. Lower bounds for the approximation of nonlinear manifolds. *Trans. Amer. Math. Soc.*, 133:167–178, 1968.
- [153] Dan E. Willard. Lower bounds for the addition-subtraction operations in orthogonal range queries and related problems. *Inform. Comput.*, 82(1):45–64, 1989.
- [154] A. C. Yao. A lower bound to finding convex hulls. *J. ACM*, 28:780–787, 1981.

- [155] A. C. Yao. On the complexity of maintaining partial sums. *SIAM J. Comput.*, 14:277–288, 1985.
- [156] A. C. Yao. Lower bounds for algebraic computation trees with integer inputs. *SIAM J. Comput.*, 20:655–668, 1991.
- [157] A. C. Yao and R. L. Rivest. On the polyhedral decision problem. *SIAM J. Comput.*, 9:343–347, 1980.
- [158] Andrew Chi-Chih Yao. Decision tree complexity and Betti numbers. In *Proc. 26th Annu. ACM Sympos. Theory Comput.*, pages 615–624, 1994.
- [159] Andrew Chi-Chih Yao. Algebraic decision trees and Euler characteristics. *Theor. Comput. Sci.*, 141(1–2):133–150, 1995.
- [160] C. K. Yap. A geometric consistency theorem for a symbolic perturbation scheme. *J. Comput. Syst. Sci.*, 40:2–18, 1990.
- [161] G. M. Ziegler. *Lectures on Polytopes*, volume 152 of *Graduate Texts in Mathematics*. Springer-Verlag, 1994.

Electronic addresses are given for preliminary unpublished versions of some references. These addresses are current as of July 1996, but may change at any time. Once these works are formally published, electronic copies may no longer be available.

Index of Notation

| | | |
|---------------------|---|--------|
| \mathcal{A} | an algorithm | 56, 83 |
| $A \sqsubseteq B$ | A is simpler than B | 97 |
| \mathcal{C} | a cell in an arrangement of hyperplanes | 57, 96 |
| Δ | the maximum outdegree of a partition graph | 81 |
| ε | a generic infinitesimal | 49 |
| | a sufficiently small positive real number | 25, 72 |
| | an arbitrarily positive constant (in an asymptotic time bound) | 62 |
| f^* | the dual flat of f | 94 |
| $I(P, H)$ | the number of incidences between points P and hyperplanes H | 69 |
| $I_d(n, m)$ | maximum number of incidences between n points and m hyperplanes in \mathbb{R}^d | 69 |
| $i \perp j$ | i and j are relatively prime | 72 |
| $I(P, H)$ | the number of incidences between points P and hyperplanes H | 100 |
| $K(\varepsilon)$ | an extension of the ordered field K | 49 |
| \tilde{K} | the real closure of the ordered field K | 49 |
| $\log^* n$ | iterated logarithm of n | 11 |
| $M(P, H)$ | the relative orientation matrix of points P and hyperplanes H | 68 |
| $\mu(P, H)$ | minimum monochromatic cover size of points P and hyperplanes H | 69 |
| $\mu_d(n, m)$ | worst case monochromatic cover size for n points and m hyperplanes in \mathbb{R}^d | 69 |
| $\mu_d^\circ(n, m)$ | worst case monochromatic cover size for n points and m hyperplanes in \mathbb{R}^d with no incidences | 69 |
| $[n]$ | the integers $\{1, 2, \dots, n\}$ | 72 |
| $n^{\underline{a}}$ | falling factorial power: $n!/(n-a)!$ | 58 |
| $\Omega(\cdot)$ | asymptotic lower bound | 2 |

| | | |
|---|--|-----|
| $\omega_d(t)$ | the d -dimensional weird moment curve: $(t, t^2, \dots, t^{d-1}, t^{d+1})$ | 20 |
| ϕ | a fixed linear expression in r variables | 50 |
| $\varphi(n)$ | the Euler totient function | 72 |
| $\Phi \leq \Pi$ | Φ is a face of Π | 94 |
| $\pi_r(P, H)$ | minimum r -polyhedral cover size of points P and hyperplanes H | 100 |
| $\hat{\pi}_{d,r}(n, m)$ | worst case r -polyhedral cover size for monochromatic configurations of n points and m hyperplanes in \mathbb{R}^d | 101 |
| $\pi_{d,r}^\circ(n, m)$ | worst case r -polyhedral cover size of n points and m hyperplanes in $\mathbb{R}\mathbb{P}^d$ with no incidences | 100 |
| Π^* | the dual of the projective polyhedron Π | 95 |
| $\text{proj}_f(X)$ | the projection of a set X by a flat f | 95 |
| \mathcal{Q}_A | the set of query polynomials used by algorithm \mathcal{A} | 56 |
| $\mathbb{R}(\varepsilon_1, \varepsilon_2, \varepsilon_3)$ | a tower of field extensions of the reals | 49 |
| $\mathbb{R}\mathbb{P}^d$ | d -dimensional real projective space | 94 |
| \mathcal{R}_v | the set of query regions associated with a node v in a partition graph | 81 |
| σ | the inner product doubling map from \mathbb{R}^3 to \mathbb{R}^6 | 80 |
| σ_d | the inner product doubling map from \mathbb{R}^{d+1} to $\mathbb{R}^{\binom{d+2}{2}}$ | 101 |
| $\text{span}(X)$ | the projective span of X | 94 |
| $\text{susp}_f(X)$ | the suspension of a set X by a flat f | 95 |
| $T_A(P, H)$ | the running time of algorithm \mathcal{A} given P and H as input | 83 |
| \mathcal{U} | an open set in $(\mathbb{R}\mathbb{P}^d)^n$ | 98 |
| $\#W$ | number of connected components of a set W | 5 |
| $\zeta(P, H)$ | minimum zero cover size of points P and hyperplanes H | 69 |
| $\zeta_d(n, m)$ | worst case zero cover size for n points and m hyperplanes in \mathbb{R}^d | 69 |

Index

- 3SUM-hard, 27–28, 59–60
- Abbott, Edwin, 46
- adversary, 12
- adversary argument, 4–5, 16–18, 21, 35, 51, 58, 65–67
- Agarwal, Pankaj, 38, 62, 107
- algebraic computation tree, *see* models of computation
- algebraic decision tree, *see* models of computation
- Amato, Nancy, 32, 38
- Amenta, Nina, 39
- Anderson, Laurie, iii
- apex, 94, 96
- Avis, David, 32, 38
- Ben-Or, Michael, 6, 31, 32, 35, 48
- de Berg, Mark, 91
- Bern, Marshall, viii
- Bremner, David, 32
- Brönnimann, Hervé, 7, 106
- bugs in original proofs, 13, 30, 45, 60
- Chan, Timothy, 31, 38, 39
- Chand, Donald, 31, 37
- Chazelle, Bernard, 7–9, 11, 13, 31, 37, 62, 64, 71, 80, 88, 89, 106
- Chung, Fan, 68
- Chvátal, Vašek, 107
- Clarkson, Ken, 31, 37, 38, 71, 74, 92
- close pair, 72, 76
- Cole, Richard, 62
- collapsible simplex, 18, 21, 25, 27, 35, 37–39
- collapsible triangle, 16, 23, 28
- collapsible tuple, 41, 42, 44, 50, 52–55
- comparison tree, *see* models of computation
- complete bipartite subgraph, 8, 68, 106
- configuration, 50, 57, 60
- configuration space, 24, 50, 57
- conjecture, 45, 102
- convex hull, 32
- decision tree, *see* models of computation
- degenerate facet, 33
- degenerate simplex, 14, 33
- Dietzfelbinger, Martin, 48
- Dieudonné, Jean, 12
- Dobkin, David, viii, 5, 48
- Dryden, John, 61
- Drysdale, Scot, viii
- duality, 13, 16, 22, 25, 26, 38, 63, 67, 75, 81, 87, 88, 92–95, 103, 104
- Edelsbrunner, Herbert, 13, 62, 71, 74
- edge
 - of a partition graph, 81
 - of a polytope, 33
- Einstein, Albert, 39
- elementary formula, 49, 56, 97–98
- Eppstein, David, viii
- Erdős, Pál, 64, 68, 71
- exercises for the reader, 25, 67, 71
- face, 94
 - dimension of, 94
- face lattice, 94, 95
- facet, 33
- flat, 94
 - dual, 94, 98
- Fredman, Michael, 7, 47, 48, 58, 71
- Fredman/Yao semigroup arithmetic model, *see* models of computation

- Fukuda, Komei, 38
 Füredi, Zoltán, 22
- Gaiman, Neil, vii
 Gajentaan, Anka, 27
 Gale, David, 33
 Goodrich, Mike, viii
 Grünbaum, Branko, 23, 32
 Graham, Ron, 31
 green, 57
 Guibas, Leo, 13, 71, 74
- Heisel, Carl Theodore, 30
 Hershberger, John, 62
 homogeneous coordinates, 24, 42, 65, 80, 97, 101
 Hong, Se June, 31
 Hopcroft, John, 62
 hyperplane arrangement, 14, 57, 58, 94, 96
- incidence graph, 8, 68, 75, 106
 index, 124–127
 infinitesimal, 49
 information theory, 3, 6
- Jacobi, Carl Gustav Jakob, 21
- Kapur, Sham, 31, 37
- Lee, Der-Tsai, 13
 van Leeuwen, Jan, 13
 Lem, Stanislaw, 60
 linear decision tree, *see* models of computation
 Lipton, Richard, 5, 48
 lonely, 75
 loosely monochromatic, 76
- Maass, Wolfgang, 48
 Matoušek, Jiří, 8, 38, 62, 80, 88, 92, 106
 Mehlhorn, Kurt, vii
 Meyer auf der Heide, Friedhelm, 48
 Milnor, John, 6
 Mněv, Nikolai, 29
 models of computation, 2
 algebraic computation tree, 6, 14, 31
 algebraic computation trees, 107
 algebraic decision tree, 5–7, 14, 24, 31, 91
 limitations, 6
 algebraic decision trees, 107
 comparison tree, 4
 decision tree, 3–7, 14
 direct query decision tree, 47
 group arithmetic, 8
 integer RAM, 28
 linear decision tree, 5, 47, 50
 partitioning algorithms, 63, 81–89
 containment shortcut, 86–87, 105
 nondeterminism of, 82
 polyhedral, 102–106
 trivial, 85, 104, 105
 unbounded, 86
 vs. real algorithms, 88–89
 pointer machine, 9
 r-linear decision tree, 47, 50–59
 semigroup arithmetic, 7–8, 64, 80, 106
 limitations, 8
 moment curve, 20
 monochromatic, 68, 101
 monochromatic cover, 68–80, 84, 85
 size, 68
- n^2 -hard, *see* 3SUM-hard
 nondegenerate with respect to a query, 24, 37, 42
 nonuniform algorithm, 29, 58–59
 notation, 3, 11, 20, 62, 69, 72, 91, 94–96, 99–100, 122–123
 NP-hard, 6, 29, 32, 39, 60
- O’Rourke, Joseph, vii, 13
 obvious, 4, 5, 8, 21, 28, 59, 65, 106
 Oleňnik, Olga Arsenievna, 6
 open problems, 5, 28, 38–39, 45–46, 59–60, 67, 79, 89–91, 106–107
 order type, 14
 ordered field, 49
 orientation of a simplex, 14
 orientation test, *see* primitives, sidedness
 query

- output size, 2, 9, 31–32
- Overmars, Mark, 27, 91
- Pach, János, 7, 106
- Palásti, Ilona, 22
- partition graph, 81
 - as a range searching data structure, 107
 - level of node in, 82
 - polyhedral, 102
 - primal and dual edges, 84
 - primal and dual nodes, 81, 88, 102
- partitioning algorithms, *see* models of computation
- partner, 82, 103
- Petrovskiĭ, Ivan Goergievič, 6
- Plato, 61
- polyhedral cover, 99–102, 105
 - size, 99
- polyhedral minor, 99
- polyhedral partitioning algorithm, *see* models of computation
- polyhedron, *see* projective polyhedron
- polytope, 32
 - face of, 32
 - projective, 94
- Preparata, Franco, 31
- primitives
 - algebraic query, 24, 66
 - allowable query, 23–26, 37, 59
 - examples, 25–26
 - circularly allowable query, 42
 - direct query, 47
 - incircle query, 40
 - insphere query, 26, 40
 - line query, 65
 - point query, 65
 - relative orientation query, 64
 - sidedness query, 13, 21, 23
 - tuple comparison, 19
 - used by real convex hull algorithms, 37–38
- problems
 - 20 questions, 3
 - 3SUM, 27, 53
 - affine degeneracy, 13–30, 47
 - arbitrary, 20–26
 - input in convex position, 21
 - nonvertical, 15–18
 - circular degeneracy
 - arbitrary, 41–42
 - proper, 41
 - convex hull, 31–39
 - cyclic overlap, 90
 - element uniqueness, 5, 28, 47
 - extreme points, 39, 107
 - halfspace emptiness, 67
 - Hopcroft’s problem, 47, 62–67, 82–86, 105–106
 - counting version, 80, 85–86
 - linear programming, 107
 - linear satisfiability, 19, 47–60
 - MAX, 4
 - median selection, 4
 - minimum measure simplex, 26
 - range emptiness, 8
 - offline halfspace, 92–93, 102–105
 - range reporting, 9
 - orthogonal, 9
 - simplex, 9
 - range searching, 7–8
 - dynamic halfplane, 7
 - dynamic orthogonal, 7, 8
 - halfspace, 7, 106
 - offline halfplane, 8
 - offline orthogonal, 8
 - offline simplex, 8, 64, 106
 - orthogonal, 7
 - simplex, 7, 64
 - SEPARATOR2, 28
 - set intersection, 63
 - set membership, 5–7
 - sorting, 3
 - sorting $X + Y$, 48, 59
 - spherical degeneracy, 47
 - arbitrary, 45
 - proper, 42–45
 - projection, 95, 98
 - projective polyhedron, 94
 - dual, 95

- projective polytope, *see* polytope, projective
- projective transformation, 24, 37, 92, 94
- proper face, 94
- proper spherical degeneracy, 40
- pseudoline arrangement, 29
- quasi-simplicial polytope, 33
- query, 3
- query polynomial, 5, 24
- query region, 81, 102
- r-linear decision tree, *see* models of computation
- r-separable, 96
- Ramos, Edgar, 32, 38
- randomization, viii
- real closed field, 49
- real closure, 49
- red, 57
- relative orientation matrix, 68
- relatively collapsible tuple, 56–57
- ridge, 33, 38
- Rosenberg, Burton, 8, 9, 106
- Rucker, Rudy, 91
- Schur, Issai, 21
- Schwarzkopf, Otfried, vii, 38, 91
- Seidel, Miriam, vii
- Seidel, Raimund, vii, 13, 25, 31, 32, 37, 38, 42, 62, 89
- semialgebraic, 6, 50
- semigroup arithmetic model, *see* models of computation
- separates, 96
- separation of lines and pseudolines, 29
- Sharir, Micha, 13, 62, 71, 74
- Shor, Peter, 31, 37
- simple minor, 69
- simpler, 39, 97
- simplicial polytope, 33
- Snoeyink, Jack, viii, 31, 38, 62
- Spencer, Joel, 68
- Steele, J. Michael, 6, 48
- Steiger, Bill, 29, 59
- Stolfi, Jorge, 37
- Streinu, Ileana, 29, 59
- supporting hyperplane, 32, 94
- suspension, 95, 99
- Swart, Garret, 37
- Sylvester, James, 22
- Székeley, László, 71
- Szemerédi, Endre, 71
- Tarján, T. G., 68
- Tarjan, Robert, 9
- Tarski, Alfred, 48–49
- Thom, René, 6
- Transfer Principle, 49, 56
- Trotter, William T., 71
- Tuza, Zsolt, 68
- vague “conjecture”, 28–30, 39
- Vandermonde matrix, 52, 77
 - almost, 20
- vertex
 - of a polytope, 33
- weird moment curve, 20–22, 33–36
- Welzl, Emo, vii, viii, 62, 71, 74
- Whittlesey, Kim, viii
- Willard, Fred, 8
- Yao, Andrew Chi-Chih, 5, 7, 31, 32, 48
- Yap, Chee, 31, 38, 62
- yellow pig, 17
- zero cover, 68
- Zhuangzi (Chuang-tsu), 12
- Ziegler, Günter, 32, 39, 94, 107