

Admin: Hw: Don't forget to tell Gradescope your teammates!
 Hw1 is graded — regrade requests due in 1 week
 Hw2 swap two \leftrightarrow three or quack \leftrightarrow steque
 Hw3 out tonight due next Monday
 No more late registration

Concatenation for deques with random access

↳ Doubly-linked lists ↳ Array list with resizing

[HEAD] [ACHE] $\xrightarrow{\text{concat}}$ [HEADACHE]

[IM] [POSSIBLE] [KINGS] [MEN]

Concat(A, B):

if $A.\text{num} < B.\text{num}$
 while A is not empty
 B.Push(A.Pull())
 else
 while B is not empty
 A.Shove(B.Pop())

total # items in all deques
 \downarrow
 $\Theta(n)$ time worst case

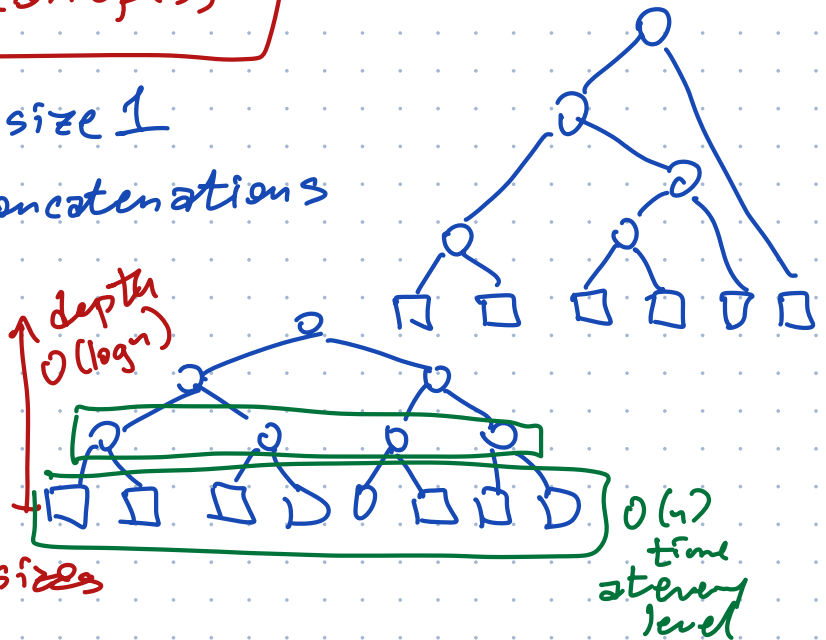
Start with n deques of size 1
 and perform $n-1$ Concatenations

How long?

Crude: $O(n^2)$

Balanced: $O(n \log n)$

$O(\log n)$ sizes



$\lceil \log_2 n \rceil = \# \text{ times we divide by } 2 \text{ to go from } n \text{ to } \leq 1$

$$\log_b n = \frac{\log n}{\log b}$$

$$\lg n = \log_2 n$$

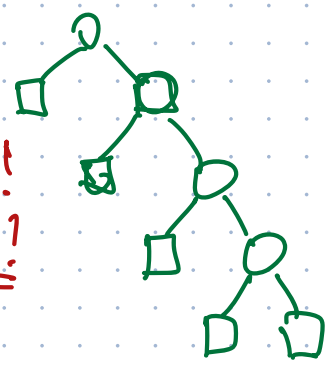
Every level: concat $\left(\frac{n}{2^{k+1}}\right)$ deque pairs
 each length 2^k
 each takes $2 \cdot 2^k$ indels

indels = n

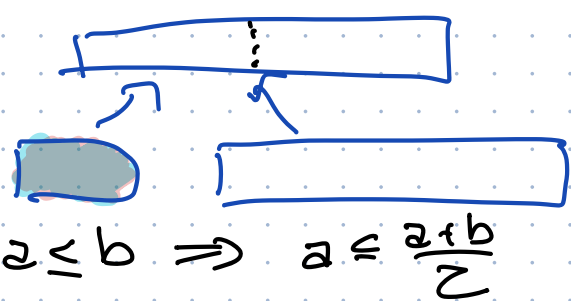
Arbitrary: $O(n \log n)$

~~Best?~~

$O(n)$!
Best!



Every item: count transfers
 Length of deque containing x
 \geq doubles every time x transfers



\Rightarrow Each item transfers $O(\log n)$ times
 n items $\Rightarrow O(n \log n)$ time in total

Am. time for each Concat = $O(\log n)$

+ Insert

Insert(x) = Concat(New(x))

\hookrightarrow pays for all future transfers

Insert: $O(\log n)$ am. time

Concat: Zero am. time

Delete: $O(1)$ am. time

Tombstone :

Sorted array

Find(x):

→ Delete(x):

Naively $\Theta(n)$ -time

Trick: Leave x there
mark it "dead"



$O(1)$ am. time
+ Find

when half items dead, rebuild