

HW 4 due tomorrow 9pm

Anakin OI 6pm tonight

HW 5 out tomorrow am

Balanced BSTs

AVL trees $\rightarrow O(\log n)$ worst case

Scapegoat trees $\rightarrow O(\log n)$ amortized

Deletions — tombstones
+ global rebuilding

Insertions — local rebuilding
of one subtree

After inserting x :

Walk up search path

If we find a node v s.t. $v.\text{height} > \log_{\beta} v.\text{size}$
rebuild v 's subtree and halt

$1 < \beta < 2$

Claim: Insert takes $O(\log n)$ am. time

Intuition: Charge time to rebuild @ v
to insertions into v 's subtree

Define $\text{Inbalance}(v) = |v.\text{left.size} - v.\text{right.size}|$

Lemma: Just before rebuild @ v

$$\text{Inbalance}(v) = \Omega(v.\text{size})$$

Proof:

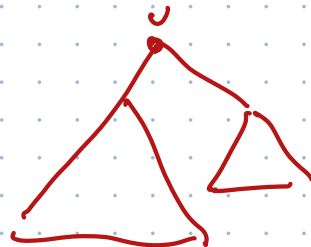
$$v.\text{height} > \log_{\beta} v.\text{size}$$

$$v.\text{left.height} \leq \log_{\beta} v.\text{left.size}$$

Math

$$v.\text{left.size} > \frac{1}{\beta} v.\text{size}$$

$$v.\text{right.size} < (1 - \frac{1}{\beta}) v.\text{size}$$



$$\text{Inbalance}(v) \geq \left(\frac{2}{\beta} - 1 \right) v.\text{size}$$

$0 < \frac{2}{\beta} < 1$

□

Augmentation

store something about v
at a new field in v .

$$v.\text{height} = \begin{cases} 0 & \text{if } v \text{ is a leaf} \\ -1 & \text{if } v = \text{NULL} \\ \max(v.\text{left}.\text{ht}, v.\text{right}.\text{ht}) + 1 & \text{o/w} \end{cases}$$

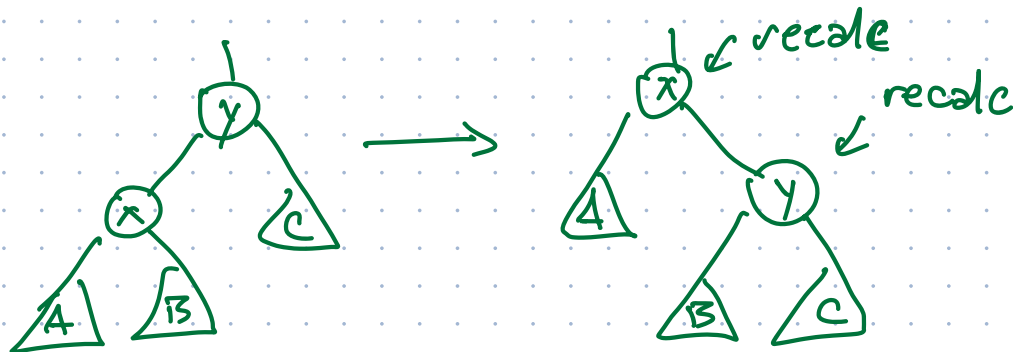
$$v.\text{size} = \begin{cases} 1 & \text{leaf} \\ 0 & \text{NULL} \\ 1 + v.\text{left}.\text{size} + v.\text{right}.\text{size} & \text{o/w} \end{cases}$$

Recursively \Leftrightarrow postorder traversal

Maintenance?

Insert: recalculate along search path
Delete:

Rotation:



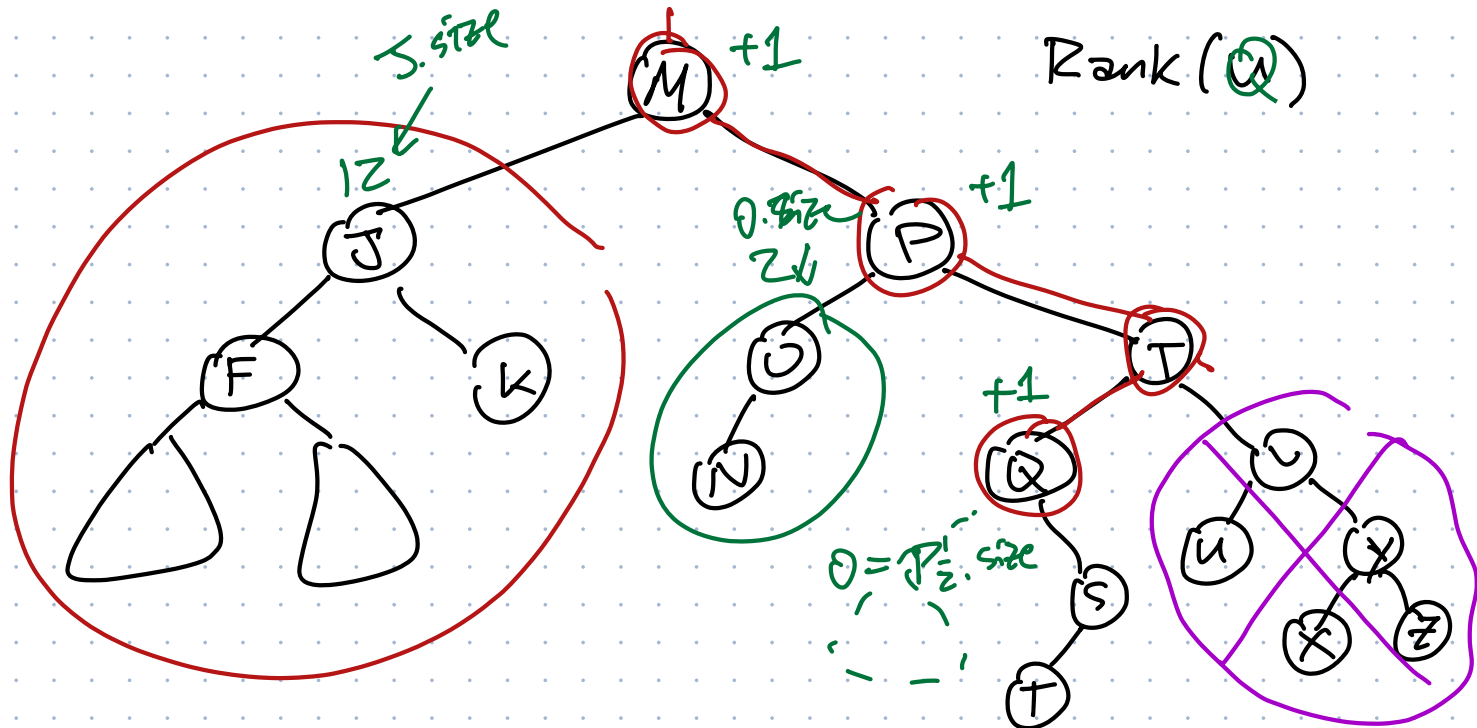
Rank(k): return the number of items
in dict that have $\text{key} \leq k$.

ABCDEFGG...XYZ

Rank(m) = 15

Select(r): return the item with rank r

Select(20) = T



Rank(u, k):

if $k = v.key$
 return $v.left.size + 1$

else if $k < v.key$
 return $Rank(v.left, k)$

else
 return $v.left.size + 1 + Rank(v.right, k)$

| | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | B | C | D | E | F | G | H | I | J | K | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
| 1 | 5 | 5 | 2 | 1 | 7 | 2 | 3 | 7 | 8 | 8 | 2 | 3 | - | - | - | - | - | - | - | - | - | - | - | - |

Max wt

Sum wt

Decomposable function

$$F(A \cup B) = F(A) \oplus F(B)$$

Rope

Split + Concat

