Splay trees

Intuition: self-adjusting

move the last search key to root using rotations

Idea: 

Find($x$): rotate at $x$ until $x$ is root
Insert($x$):

BAD IDEA

$N$ searches takes $\Omega(Nn)$ time

$\Rightarrow \mathcal{O}(n)$ time each
**Splay:**

Intuition:
- Nodes or search path reduce depth by half
- Add 1 or 2
- Other nodes might increase depth by 1 or 2.
Amortized # rotations to perform one splay
\[ \leq 1 + 3 \log_2 n \quad [\text{MAGIC!}] \]

\[ \Rightarrow \text{Am. time for FIND} = O(\log n) \]
\[ \text{INSERT} = O(\log n) \]

\[ \text{DELETE?} \quad O(\log n) \text{ am. time} \]

\[ \text{FIND}(x) \rightarrow \]

\[ \text{FIND\textsc{Pred}(x)} \]

\[ \text{SPLIT}(k) \quad O(\log n) \text{ am. time} \]

\[ \text{SPLAYTREE} \]

Rope: store set of strings
support operations:

\[ O(1) \quad \text{NewString(a)} \rightarrow \text{new string of length 1} \]
\[ O(\log n) \quad \text{Concat}(S,T) \rightarrow \text{replace } S \text{ and } T \text{ with } ST \]
\[ \text{Lookup}(S,k) \rightarrow \text{return } S[k] \]
\[ \text{Split}(S,k) \rightarrow \text{replace } S \text{ with } S[1..k] \]

Text editors
Each string in a splay tree

\[ \text{SPLAYTREE} \rightarrow \]

\[ \text{LOGAN MAGIC} \]
MAGIC:

- $O(\log n)$ am. time
- If each node $x$ is accessed $t(x)$ times
  
  $O(\log \frac{T}{t(x)})$ where $T$ = total # accesses

  Static optimality

  Pick Favorite node $F$ "Finger"
  
  $O(\log \text{dist}(F,x))$

  $D =$ #distinct items accessed since last access to $x$

  $O(\log D)$

Conjecture:

Within $O(1)$ factor of best dynamic BST