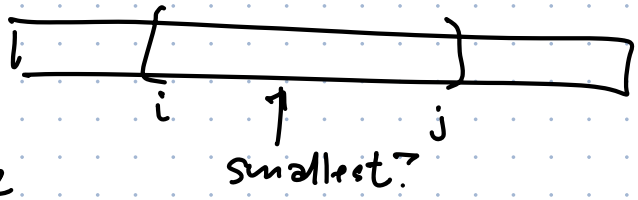
HW7 due tomorrow

HW8 due next Tue

No live lecture April 8 — HW9 out as usual

Last Mon April — joint with SigMA — Hashlife

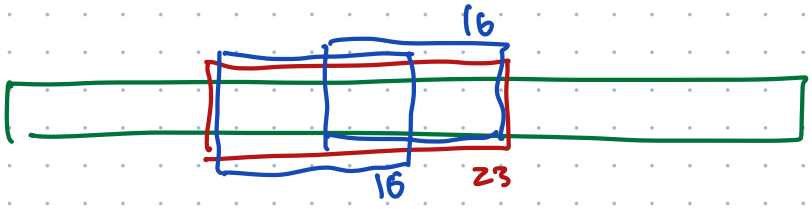
Range Minimum Query



O(n) space O(log n) time

$O(n^2)$ space O(1) time — lookup table

store answers to every query



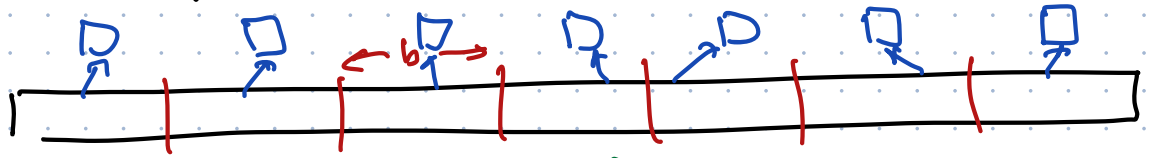
O(n log n) space O(1) time sparse table

preprocess ranges of lengths 1, 2, 4, 8, 16, ...

query = 2 lookups

Indirection / Blocking

Split input array into $\frac{n}{b}$ blocks of length b



Short query:

↓

Prep each block

Long query:

Prep block minima

① For each block:
Prep into a RMQ structure

② Build array of $\frac{n}{b}$ block minima
Prep into a RMQ structure

Space = $O(b \log b) \times \frac{n}{b} = O(n \log b)$

$O(\frac{n}{b} \log \frac{n}{b})$

Short query → one ① query
Long query → ≤2 ① query + 1 ② query
} $O(1)$ time

$b = \Theta(\log n) \Rightarrow$ space $= O(n \log \log n)$

Two levels of indirection:

space $= O(n \log\log\log n)$ Query time $= O(1)$

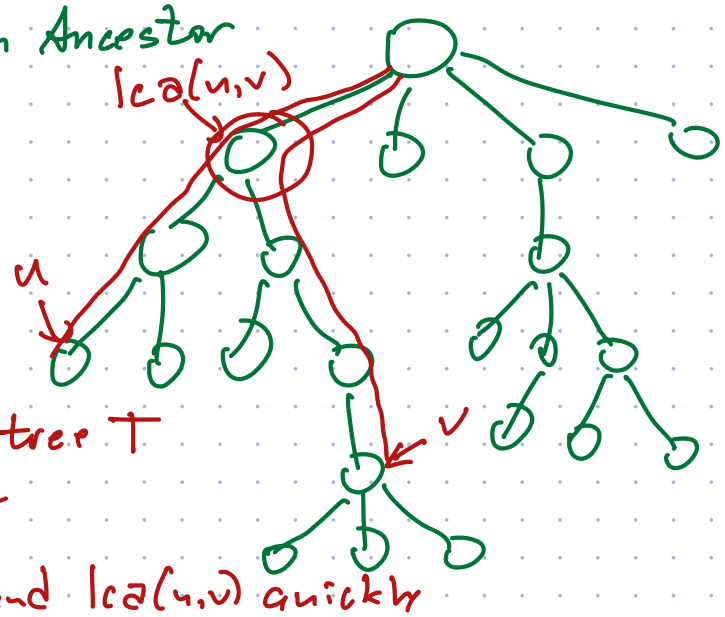
Recursively k levels:

space $= O(kn + n \underbrace{\log\log\cdots\log}_{k} n)$

query time $= O(k)$

Set $k = \log^* n = \begin{cases} 1 & \text{if } n \le 2 \\ 1 + \log^* \log n & \text{o/w} \end{cases}$

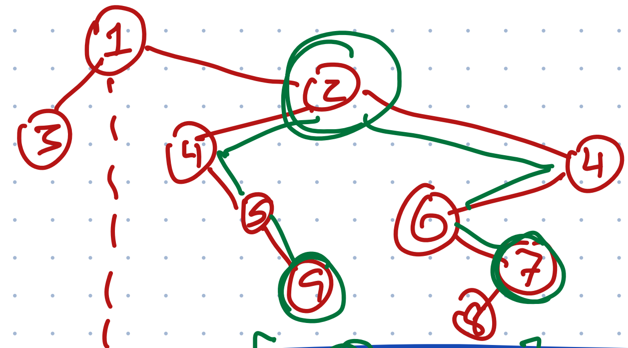
Least Common Ancestor



Given rooted tree T
Prep it
Given u, v
we can find lca(u,v) quickly

RMQ → LCA

Cartesian tree



A: [3 1 4 5 [9 2 6 8 7] 4 . . . .]

① inorder → A

② min-heap

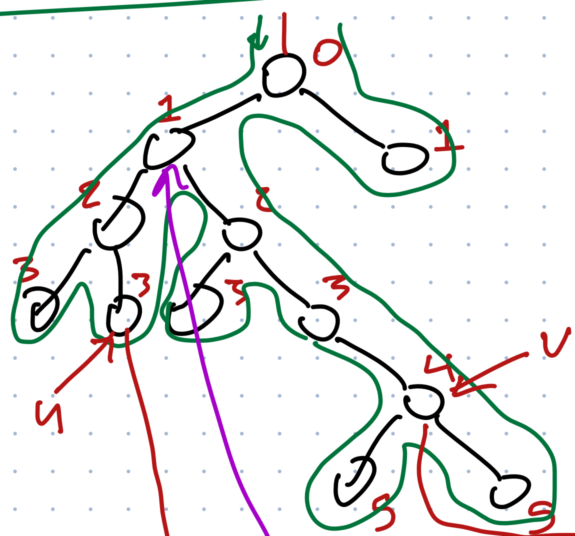
index in-order

min in $A[i..j]$ is $LCA(v_i, v_j)$

LCA → RMQ (±1)

① compute depths

② Euler tour



0 1 2 3 2 [3 2 1 2 3 2 3 4 5 4] 5 4 3 2 1 0 1 0

Indirection

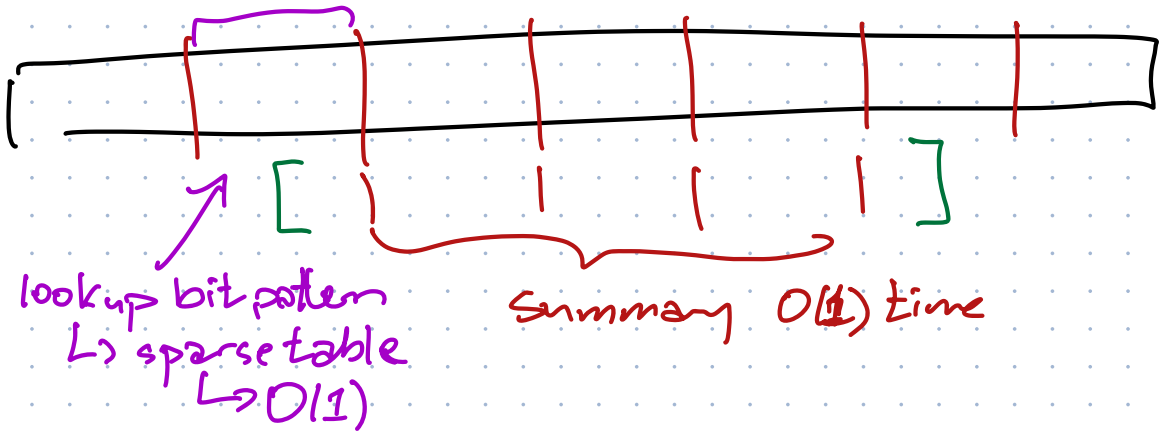
Four Russians ⇒ [↑↓↑↑↑↓↑↓] (size b) ← 2^b different patterns

Build 'em all!

$O(2^b \cdot b \log b)$ space

⇓

$b = \frac{1}{2}\log_2 n$ $\quad O(\sqrt{n}\log n \log\log n) = o(n)$



O(n) space O(1) time