

Next week: Joint with SIGMA — Hashlife

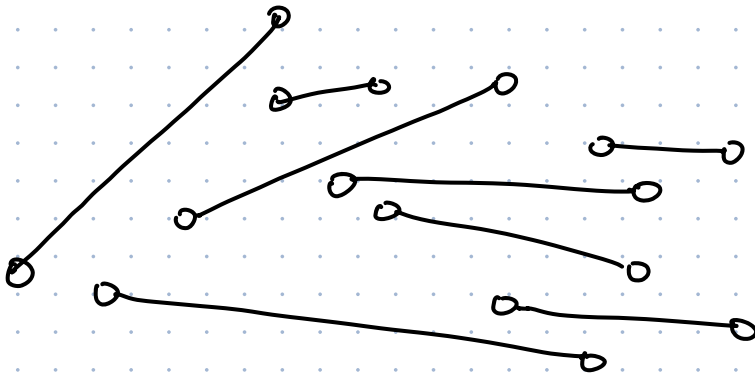
MW 10 due next Tue

≥ 20 to get ti

Computational geometry

Sweep-line algorithm — treat one dimension as time

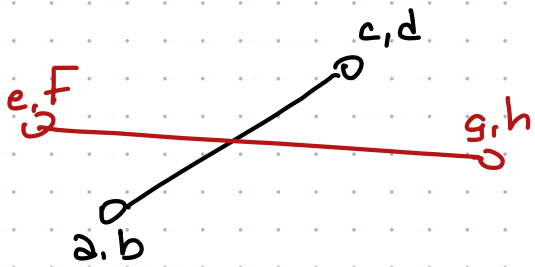
Given a set S of line segments in the plane
Do any two intersect?



Sweep a vertical line l across plane

Maintain description of $S \cap l$ at all times

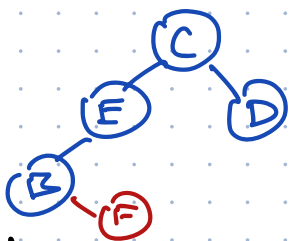
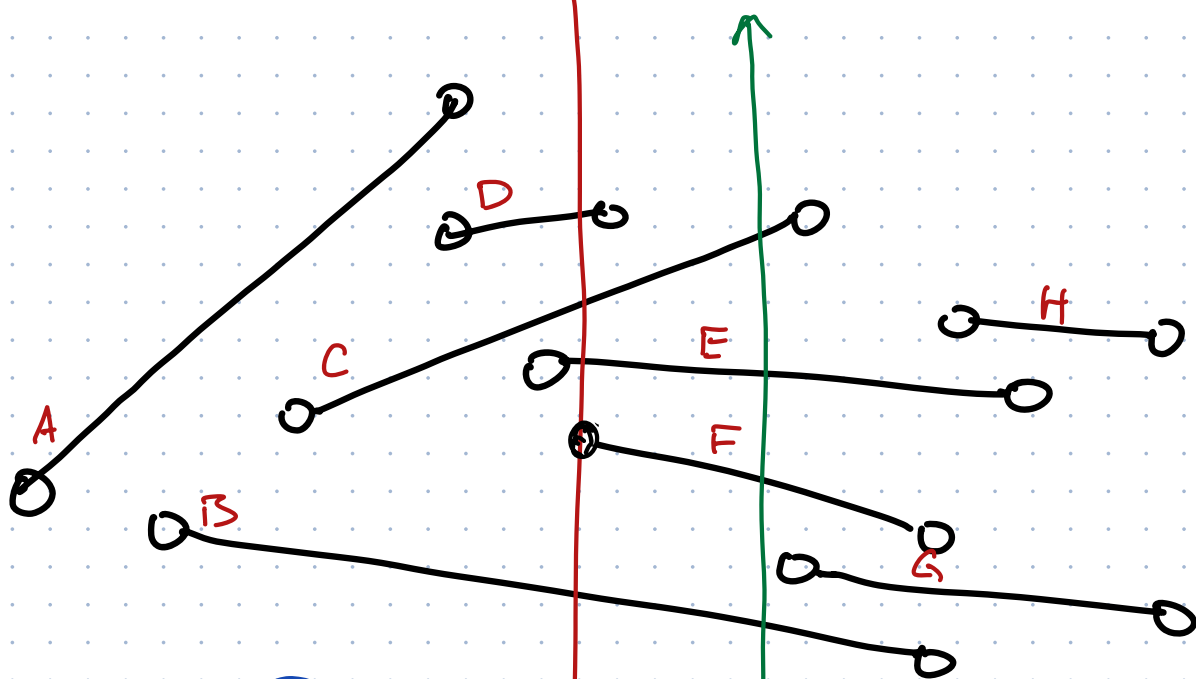
Brute force: $\Theta(n^2)$ time



test 2 segments in $O(1)$ time

$$\begin{vmatrix} 1 & a & b \\ 1 & c & d \\ 1 & e & f \end{vmatrix} \cdot \begin{vmatrix} 1 & a & b \\ 1 & c & d \\ 1 & g & h \end{vmatrix} < 0$$

$$\begin{vmatrix} 1 & a & b \\ 1 & e & f \\ 1 & g & h \end{vmatrix} \cdot \begin{vmatrix} 1 & c & d \\ 1 & e & f \\ 1 & g & h \end{vmatrix} < 0$$



B E C D

B I F E C

left endpoint \rightarrow insertion
 right endpoint \rightarrow deletion

\leftarrow store sequence of segments in order along l .

Store sequence in a balanced BST

"Comparison" = compare y-coordinates

Intersecting segments must be adjacent in sweep-line sequence just before they cross.

Insert(s): Test s and pred(s)
 Test s and succ(s)

Delete(s): Test pred(s) and succ(s)

Sort x-coords.

n left endpoints $\times O(\log n)$ time

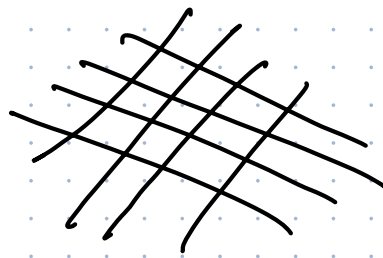
n right endpoints $\times O(\log n)$ time

$\left. \begin{array}{l} \\ \\ \end{array} \right\} \underline{\underline{O(n \log n)}}$

Count # intersecting pairs

$O((n+k) \log n)$ time

where $k = \# \text{ints}$



When we find intersection, schedule a new crossing event

↑
priority queue

n endpoints $\underbrace{\text{In, Del}}_{\text{BST}}, \text{Crossing tests} \xrightarrow{?} \text{Insert in PQ}$

k crossings: $\text{ExMin}, \underbrace{\text{InDel}}_{\text{BST}}, \text{Crossing} \xrightarrow{?} \text{Insert PQ}$