1. **Randomized Edge Cuts**

   We will randomly partition the vertex set of a graph G into two sets S and T. The algorithm is to flip a coin for each vertex and with probability $1/2$, put it in S; otherwise put it in T.

   (a) Show that the expected number of edges with one endpoint in S and the other endpoint in T is exactly half the edges in G.

   (b) Now say the edges have weights. What can you say about the sum of the weights of the edges with one endpoint in S and the other endpoint in T?

2. **Skip Lists**

   A *skip list* is built in layers. The bottom layer is an ordinary sorted linked list. Each higher layer acts as an "express lane" for the lists below, where an element in layer i appears in layer $i + 1$ with some fixed probability p.

```
1
1-----4---6
1---3-4---6-----9
1-2-3-4-5-6-7-8-9-10
```

   (a) What is the probability a node reaches height h.

   (b) What is the probability any node is above $c \log n$ (for some fixed value of c)?
   Compute the value explicitly when $p = 1/2$ and $c = 4$.

   (c) To search for an entry x, scan the top layer until you find the last entry y that is less than or equal to x. If $y < x$, drop down one layer and in this new layer (beginning at y) find the last entry that is less than or equal to x. Repeat this process (dropping down a layer, then finding the last entry less than or equal to x) until you either find x or reach the bottom layer and confirm that x is not in the skip list. What is the expected search time?

   (d) Describe an efficient method for insertion. What is the expected insertion time?

3. **Clock Solitaire**

   In a standard deck of 52 cards, put 4 face-down in each of the 12 'hour' positions around a clock, and 4 face-down in a pile in the center. Turn up a card from the center, and look at the number on it. If it's number x, place the card face-up next to the face-down pile for x, and turn up the next card in the face-down pile for x (that is, the face-down pile corresponding to hour x). You win if, for each Ace $\leq x \leq$ Queen, all four cards of value x are turned face-up before all four Kings (the center cards) are turned face-up.

   What is the probability that you win a game of Clock Solitaire?