1. Prove that the following problem is NP-hard.

   SETCOVER: Given a collection of sets $\{S_1, \ldots, S_m\}$, find the smallest sub-collection of $S_i$'s that contains all the elements of $\bigcup_i S_i$.

2. Given an undirected graph $G$ and a subset of vertices $S$, a *Steiner tree* of $S$ in $G$ is a subtree of $G$ that contains every vertex in $S$. If $S$ contains every vertex of $G$, a Steiner tree is just a spanning tree; if $S$ contains exactly two vertices, any path between them is a Steiner tree.

   Given a graph $G$, a vertex subset $S$, and an integer $k$, the *Steiner tree problem* requires us to decide whether there is a Steiner tree of $S$ in $G$ with at most $k$ edges. Prove that the Steiner tree problem is NP-hard. *[Hint: Reduce from* VERTEXCOVER*, or* SETCOVER*, or* 3SAT*.]*

3. Let $G$ be a directed graph whose edges are colored red and white. A *Canadian Hamiltonian path* is a Hamiltonian path whose edges are alternately red and white. The CANADIANHAMILTONIANPATH problem ask us to find a Canadian Hamiltonian path in a graph $G$. (Two weeks ago we looked for Hamiltonian paths that cycled through colors on the *vertices* instead of edges.)

   (a) Prove that CANADIANHAMILTONIANPATH is NP-Complete.

   (b) Reduce CANADIANHAMILTONIANPATH to HAMILTONIANPATH.