

1. A *longest common subsequence* of a set of strings $\{A_i\}$ is a longest string that is a subsequence of A_i for each i . For example, `alrit` is a longest common subsequence of strings

`algorithm` and `altruistic`.

Given two strings $A[1..n]$ and $B[1..n]$, describe and analyze a dynamic programming algorithm that computes the length of a longest common subsequence of the two strings in $O(n^2)$ time.

2. Describe and analyze a dynamic programming algorithm that computes the length of a longest common subsequence of three strings $A[1..n]$, $B[1..n]$, and $C[1..n]$ in $O(n^3)$ time. [Hint: Try **not** to use your solution to problem 1 directly.]
3. A *lucky-10 number* is a string $D[1..n]$ of digits from 1 to 9 (no zeros), such that the i -th digit and the last i -th digit sum up to 10; in another words, $D[i] + D[n - i + 1] = 10$ for all i . For example,

3141592648159697 and 11599

are both lucky-10 numbers. Given a string of digits $D[1..n]$, describe and analyze a dynamic programming algorithm that computes the length of a longest lucky-10 subsequence of the string. [Hint: Try to use your solution to problem 1 **directly**.]

4. **To think about later:** Can you solve problem 1 in $O(n)$ space?