"CS 374" Fall 2014 ✧ Homework 11

Due Tuesday, December 9, 2014 at noon

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1. Recall that w^R denotes the reversal of string w; for example, $\texttt{TURING}^R = \texttt{GNIRUT}$. Prove that the following language is undecidable.

$$\textsc{RevAccept} := \left\{ \langle M \rangle \;\middle|\; M \text{ accepts } \langle M \rangle^R \right\}$$

2. Let M be a Turing machine, let w be an arbitrary input string, and let s be an integer. We say that ***M accepts w in space s*** if, given w as input, M accesses only the first s cells on the tape and eventually accepts.

   (a) Prove that the following language is decidable:

$$\left\{ \langle M, w \rangle \;\middle|\; M \text{ accepts } w \text{ in space } |w|^2 \right\}$$

   (b) Prove that the following language is undecidable:

$$\left\{ \langle M \rangle \;\middle|\; M \text{ accepts at least one string } w \text{ in space } |w|^2 \right\}$$

3. ***[Extra credit]*** For each of the following languages, either prove that the language is decidable, or prove that the language is undecidable.

   (a) $L_0 = \left\{ \langle M \rangle \;\middle|\; \text{given any input string, } M \text{ eventually leaves its } \texttt{start} \text{ state} \right\}$

   (b) $L_1 = \left\{ \langle M \rangle \;\middle|\; M \text{ decides } L_0 \right\}$

   (c) $L_2 = \left\{ \langle M \rangle \;\middle|\; M \text{ decides } L_1 \right\}$

   (d) $L_3 = \left\{ \langle M \rangle \;\middle|\; M \text{ decides } L_2 \right\}$

   (e) $L_4 = \left\{ \langle M \rangle \;\middle|\; M \text{ decides } L_3 \right\}$

CS 374 Fall 2014 ✧ Homework 11 Problem 1

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Prove that $\textsc{RevAccept} := \left\{ \langle M \rangle \;\middle|\; M \text{ accepts } \langle M \rangle^R \right\}$ is undecidable.

CS 374 Fall 2014 ✧ Homework 11 Problem 2

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(a) Prove that $\{\langle M, w\rangle \mid M \text{ accepts } w \text{ in space } |w|^2\}$ is decidable.

(b) Prove that $\{\langle M\rangle \mid M \text{ accepts at least one string } w \text{ in space } |w|^2\}$ is undecidable:

CS 374 Fall 2014 ✧ Homework 11 Problem 3

[Extra credit]

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For each of the following languages, either prove that the language is decidable, or prove that the language is undecidable.

(a) $L_0 = \{\langle M\rangle \mid \text{given any input string, } M \text{ eventually leaves its start state}\}$

(b) $L_1 = \{\langle M\rangle \mid M \text{ decides } L_0\}$

(c) $L_2 = \{\langle M\rangle \mid M \text{ decides } L_1\}$

(d) $L_3 = \{\langle M\rangle \mid M \text{ decides } L_2\}$

(e) $L_4 = \{\langle M\rangle \mid M \text{ decides } L_3\}$