

In lecture, Alex described an algorithm of Karatsuba that multiplies two  $n$ -digit integers using  $O(n^{\lg 3})$  single-digit additions, subtractions, and multiplications. In this lab we'll look at some extensions and applications of this algorithm.

1. Describe an algorithm to compute the product of an  $n$ -digit number and an  $m$ -digit number, where  $m < n$ , in  $O(m^{\lg 3 - 1}n)$  time.
2. Describe an algorithm to compute the decimal representation of  $2^n$  in  $O(n^{\lg 3})$  time. (The standard algorithm that computes one digit at a time requires  $\Theta(n^2)$  time.)
3. Describe a divide-and-conquer algorithm to compute the decimal representation of an arbitrary  $n$ -bit binary number in  $O(n^{\lg 3})$  time. [Hint: Let  $x = a \cdot 2^{n/2} + b$ . Watch out for an extra log factor in the running time.]

**Think about later:**

4. Suppose we can multiply two  $n$ -digit numbers in  $O(M(n))$  time. Describe an algorithm to compute the decimal representation of an arbitrary  $n$ -bit binary number in  $O(M(n)\log n)$  time.