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## CS 473: Undergraduate Algorithms, Spring 2009

### HBS 10

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1. Consider the following problem, called *BOX-DEPTH*: Given a set of  $n$  axis-aligned rectangles in the plane, how big is the largest subset of these rectangles that contain a common point?
  - (a) Describe a polynomial-time reduction from *BOX-DEPTH* to *MAX-CLIQUE*.
  - (b) Describe and analyze a polynomial-time algorithm for *BOX-DEPTH*. [Hint:  $O(n^3)$  time should be easy, but  $O(n \log n)$  time is possible.]
  - (c) Why don't these two results imply that  $P = NP$ ?
  
2. Suppose you are given a magic black box that can determine in polynomial time, given an arbitrary weighted graph  $G$ , the length of the shortest Hamiltonian cycle in  $G$ . Describe and analyze a polynomial-time algorithm that computes, given an arbitrary weighted graph  $G$ , the shortest Hamiltonian cycle in  $G$ , using this magic black box as a subroutine.
  
3. Prove that the following problems are NP-complete.
  - (a) Given an undirected graph  $G$ , does  $G$  have a spanning tree in which every node has degree at most 17?
  - (b) Given an undirected graph  $G$ , does  $G$  have a spanning tree with at most 42 leaves?