
CS 473: Undergraduate Algorithms, Spring 2009

HBS 11

1. You step in a party with a camera in your hand. Each person attending the party has some friends there. You want to have exactly one picture of each person in your camera. You want to use the following protocol to collect photos. At each step, the person that has the camera in his hand takes a picture of one of his/her friends and pass the camera to him/her. Of course, you only like the solution if it finishes when the camera is in your hand. Given the friendship matrix of the people in the party, design a polynomial algorithm that decides whether this is possible, or prove that this decision problem is NP-hard.

2. A boolean formula is in disjunctive normal form (DNF) if it is a disjunctions (OR) of several clauses, each of which is the conjunction (AND) of several literals, each of which is either a variable or its negation. For example:

$$(a \wedge b \wedge c) \vee (\bar{a} \wedge b) \vee (\bar{c} \wedge x)$$

Given a DNF formula give a polynomial algorithm to check whether it is satisfiable or not. Why this does not imply $P = NP$.

3. Prove that the following problems are NP-complete.
 - (a) Given an undirected graph G , does G have a spanning tree in which every node has degree at most 17?
 - (b) Given an undirected graph G , does G have a spanning tree with at most 42 leaves?