This homework is practice only. However, there will be at least one NP-hardness problem on the final exam, so working through this homework is *strongly* recommended. Students/groups are welcome to submit solutions for feedback (but not credit) in class on May 4, after which we will publish official solutions.

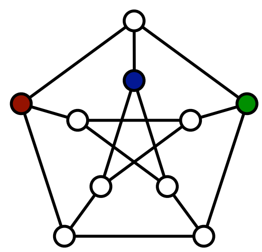
1. Recall that 3SAT asks whether a given boolean formula in conjunctive normal form, with exactly three literals in each clause, is satisfiable. In class we proved that 3SAT is NP-complete, using a reduction from CircuitSAT.

   Now consider the related problem **2SAT**: Given a boolean formula in conjunctive normal form, with exactly *two* literals in each clause, is the formula satisfiable? For example, the following boolean formula is a valid input to 2SAT:

   $$(x \vee y) \wedge (y \vee \overline{z}) \wedge (\overline{x} \vee z) \wedge (\overline{w} \vee y).$$

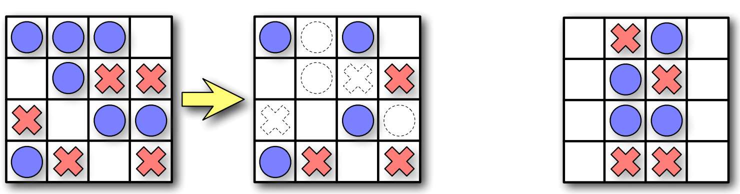
   Either prove that 2SAT is NP-hard or describe a polynomial-time algorithm to solve it. *[Hint: Recall that* $(x \vee y) \equiv (\overline{x} \rightarrow y)$*, and build a graph.]*

2. Let $G = (V, E)$ be a graph. A *dominating set* in G is a subset S of the vertices such that every vertex in G is either in S or adjacent to a vertex in S. The DominatingSet problem asks, given a graph G and an integer k as input, whether G contains a dominating set of size k. Either prove that this problem is NP-hard or describe a polynomial-time algorithm to solve it.



A dominating set of size 3 in the Peterson graph.

3. Consider the following solitaire game. The puzzle consists of an $n \times m$ grid of squares, where each square may be empty, occupied by a red stone, or occupied by a blue stone. The goal of the puzzle is to remove some of the given stones so that the remaining stones satisfy two conditions: (1) every row contains at least one stone, and (2) no column contains stones of both colors. For some initial configurations of stones, reaching this goal is impossible.



A solvable puzzle and one of its many solutions. An unsolvable puzzle.

   Prove that it is NP-hard to determine, given an initial configuration of red and blue stones, whether the puzzle can be solved.