

CS/ECE 374 A ✧ Spring 2018

☞ “Homework” 11 ☞

“Due” Tuesday, May 1, 2018

This homework is optional. However, **similar undecidability questions may appear on the final exam**, so we still strongly recommend treating at least those questions as regular homework. Solutions will be released next Tuesday as usual.

1. Let M be a Turing machine, let w be an arbitrary input string, and let s be an integer. We say that M **accepts w in space s** if, given w as input, M accesses only the first s (or fewer) cells on its tape and eventually accepts.

- * (a) Sketch a Turing machine/algorithm that correctly decides the following language:

$$\text{SQUARESPACE} = \{ \langle M, w \rangle \mid M \text{ accepts } w \text{ in space } |w|^2 \}$$

- (b) Prove that the following language is undecidable:

$$\text{SOMESQUARESPACE} = \{ \langle M \rangle \mid M \text{ accepts at least one string } w \text{ in space } |w|^2 \}$$

2. Consider the following language:

$$\text{PICKY} = \left\{ \langle M \rangle \mid \begin{array}{l} M \text{ accepts at least one input string} \\ \text{and } M \text{ rejects at least one input string} \end{array} \right\}$$

- (a) Prove that PICKY is undecidable.
- (b) Sketch a Turing machine/algorithm that *accepts* PICKY.