Write your answers in the separate answer booklet.
Please return this question sheet and your cheat sheet with your answers.

1. For each statement below, check "Yes" if the statement is ***always*** true and "No" otherwise. Each correct answer is worth +1 point; each incorrect answer is worth −½ point; checking "I don't know" is worth +¼ point; and flipping a coin is (on average) worth +¼ point. You do ***not*** need to prove your answer is correct.

   Read each statement *very* carefully. Some of these are deliberately subtle.

   (a) Every infinite language is regular.

   (b) If L is not regular, then for every string $w \in L$, there is a DFA that accepts w.

   (c) If L is context-free and L has a finite fooling set, then L is regular.

   (d) If L is regular and $L' \cap L = \varnothing$, then L' is regular.

   (e) The language $\{0^i 1^j 0^k \mid i + j + k \geq 374\}$ is not regular.

   (f) The language $\{0^i 1^j 0^k \mid i + j - k \geq 374\}$ is not regular.

   (g) Let $M = (Q, \{0, 1\}, s, A, \delta)$ be an arbitrary DFA, and let $M' = (Q, \{0, 1\}, s, A, \delta')$ be the DFA obtained from M by changing every 0-transition into a 1-transition and vice versa. More formally, M and M' have the same states, input alphabet, starting state, and accepting states, but $\delta'(q, 0) = \delta(q, 1)$ and $\delta'(q, 1) = \delta(q, 0)$. Then $L(M) \cap L(M') = \varnothing$.

   (h) Let $M = (Q, \Sigma, s, A, \delta)$ be an arbitrary NFA, and $M' = (Q', \Sigma, s, A', \delta')$ be any NFA obtained from M by deleting some subset of the states. More formally, we have $Q' \subseteq Q$, $A' = A \cap Q'$, and $\delta'(q, a) = \delta(q, a) \cap Q'$ for all $q \in Q'$. Then $L(M') \subseteq L(M)$.

   (i) For every regular language L, the language $\left\{0^{|w|} \;\middle|\; w \in L\right\}$ is also regular.

   (j) For every context-free language L, the language $\left\{0^{|w|} \;\middle|\; w \in L\right\}$ is also context-free.

2. For any language L, define

$$\textsc{StripInit0s}(L) = \left\{w \;\middle|\; 0^j w \in L \text{ for some } j \geq 0\right\}$$

   Less formally, $\textsc{StripInit0s}(L)$ is the set of all strings obtained by stripping any number of initial 0s from strings in L. For example, if L is the one-string language $\{00011010\}$, then

$$\textsc{StripInit0s}(L) = \{00011010, 0011010, 011010, 11010\}.$$

   Prove that if L is a regular language, then $\textsc{StripInit0s}(L)$ is also a regular language.

3. For each of the following languages L over the alphabet $\Sigma = \{0, 1\}$, give a regular expression that represents L ***and*** describe a DFA that recognizes L.

   (a) $\{0^n w 1^n \mid n > 1 \text{ and } w \in \Sigma^*\}$

   (b) All strings in 0^*10^* whose length is a multiple of 3.

4. The *parity* of a bit-string is 0 if the number of 1 bits is even, and 1 if the number of 1 bits is odd. For example:

$$parity(\varepsilon) = 0 \qquad parity(0010100) = 0 \qquad parity(00101110100) = 1$$

   (a) Give a *self-contained*, formal, recursive definition of the *parity* function. In particular, do ***not*** refer to # or other functions defined in class.

   (b) Let L be an arbitrary regular language. Prove that the language $EvenParity(L) := \{w \in L \mid parity(w) = 0\}$ is also regular.

   (c) Let L be an arbitrary regular language. Prove that the language $AddParity(L) := \{w \bullet parity(w) \mid w \in L\}$ is also regular. For example, if L contains the string 11100 and 11000, then $AddParity(L)$ contains the strings 111001 and 110000.

5. Let L be the language $\left\{0^i 1^j 0^k \;\middle|\; i = j \text{ or } j = k\right\}$.

   (a) ***Prove*** that L is not a regular language.

   (b) Describe a context-free grammar for L.