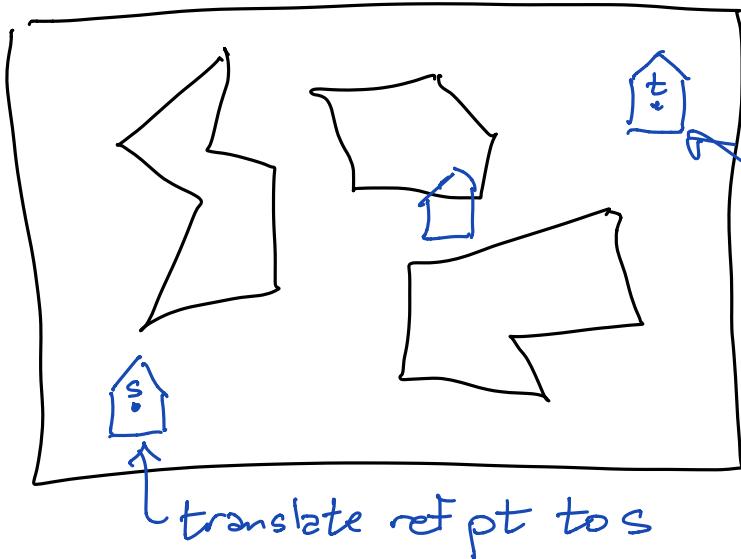


Minkowski Sums + Motion Planning



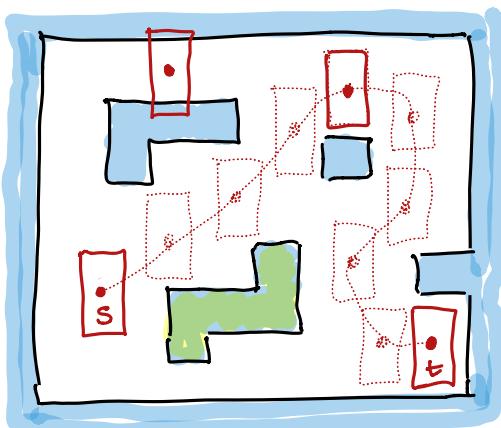
Can - "Spoon"
Talking Heads - "Radio Head"
Bonzo Dog Band - "Death Cab for Cutie"

Polygonal robot R
reference point
translate ref pt to t
Polygonal obstacles
 $P_1 \dots P_x$

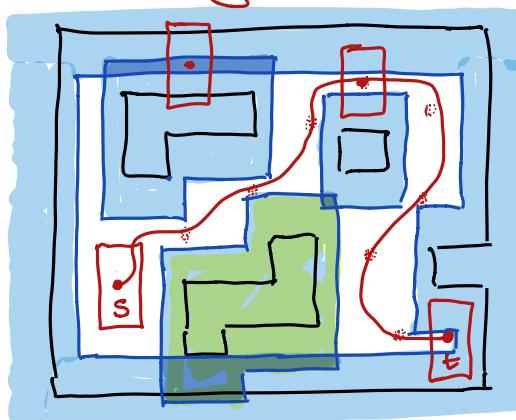
Can we translate R
from s to t
without colliding
with any P_i ?

Can we find the best path from s to t ?

Work space
Original obstacles + robot



Configuration space
= space of all robot
configurations — location of
ref. pt.



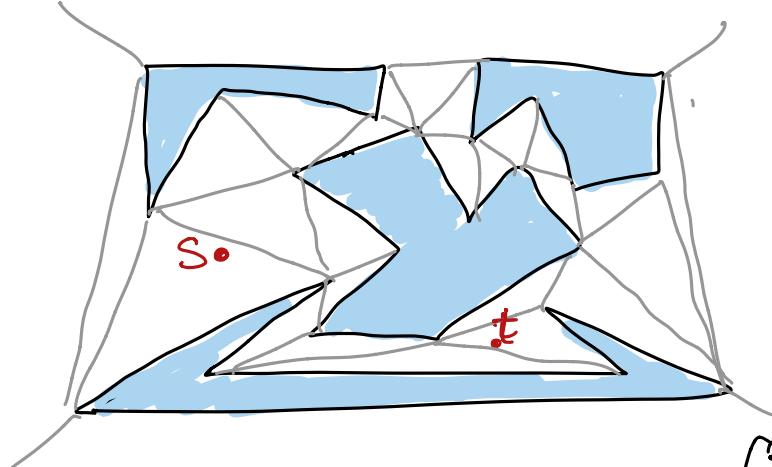
Replace robot with ref pt.

Replace obstacles with
config obstacle

= configs where robot intersects
each obstacle

There is 2 paths from s to t
in free config space
avoids all config obstacle

Robot can move from s to t
without hitting
any obstacle



Point robot:

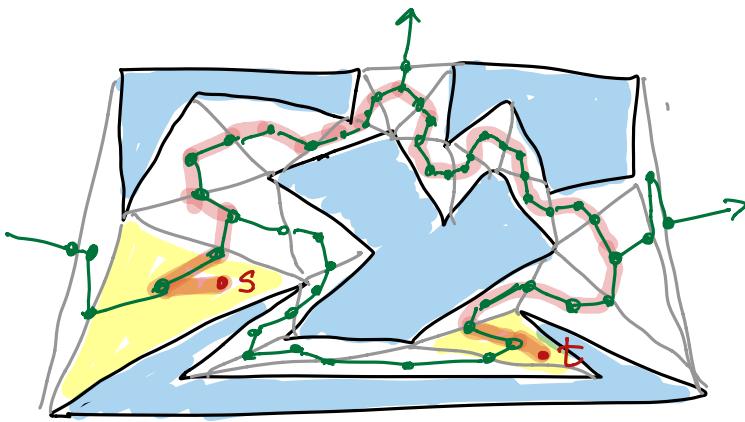
workspace = config space

① Decompose Free space F into triangulation T

② There is a path from s to t in F

iff There is a path in dual graph T^* from $\Delta(s)$ to $\Delta(t)$

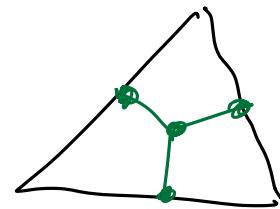
Construct the path in a road map



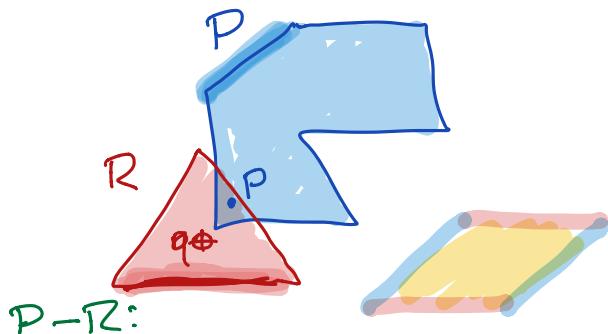
Shortest path — adapt funnels

Maximum clearance — Voronoi diagram

⋮



Polygonal robot?



For any obstacle P ,

corr. config. obstacle is

$$CP = \{q \mid \underbrace{R@q}_{\substack{\text{robot} \\ \text{translated to } q}} \cap P \neq \emptyset\}$$

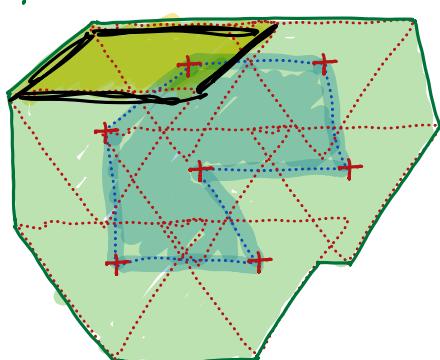
$$R@q = R + q = \{r + q \mid r \in R\}$$

Minkowski Sum

$$CP = P - R = \{p - r \mid p \in P \text{ and } r \in R\}$$

$$= P + (-R)$$

↑ rotate R half turn around origin



Decompose by Features: $|P|=m$ $|Q|=n$

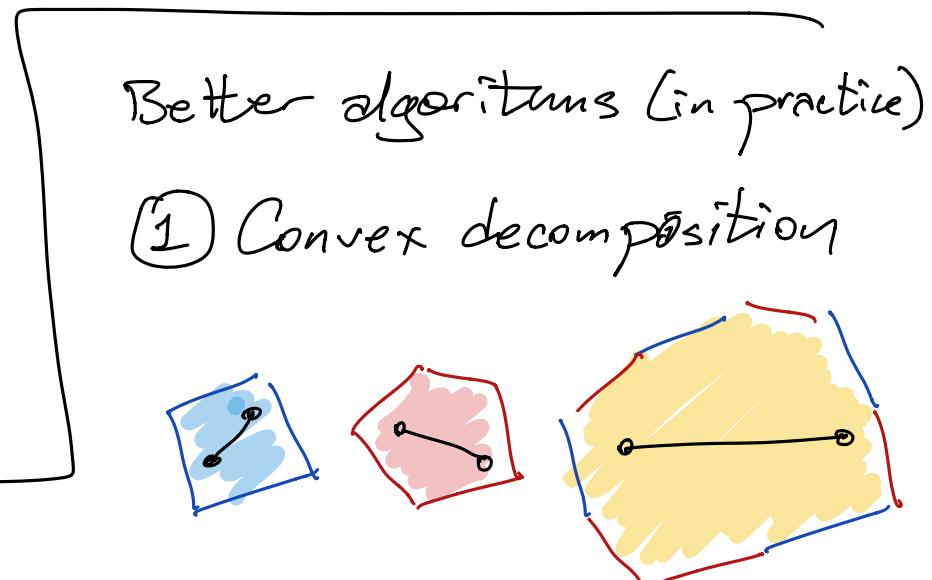
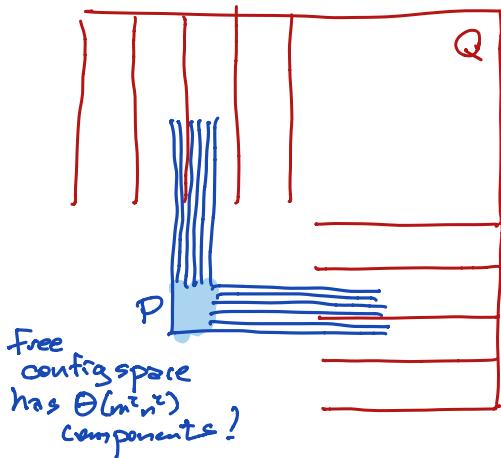
Every point in $P+Q$ is one of the following:

- vertex of P + int pt of Q → copies of Q
- edge pt of P + edge pt of Q ⇒ rhombuses!
- int pt of P + vertex of Q → copies of P

$\Rightarrow P+Q$ is the union of $O(mn)$ polygons
with total of $O(mn)$ vertices

\Rightarrow Total complexity of $P+Q$ is $O(m^2n^2)$

we can build $P+Q$ in $O(m^2n^2 \text{ time})$
This is tight in worst case!



IF P and Q are convex, then

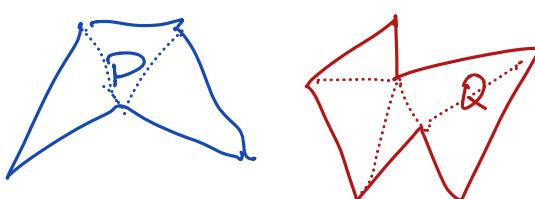
$P+Q$ is also convex

computed by merging edges of P and Q
by direction

$O(m+n)$ time

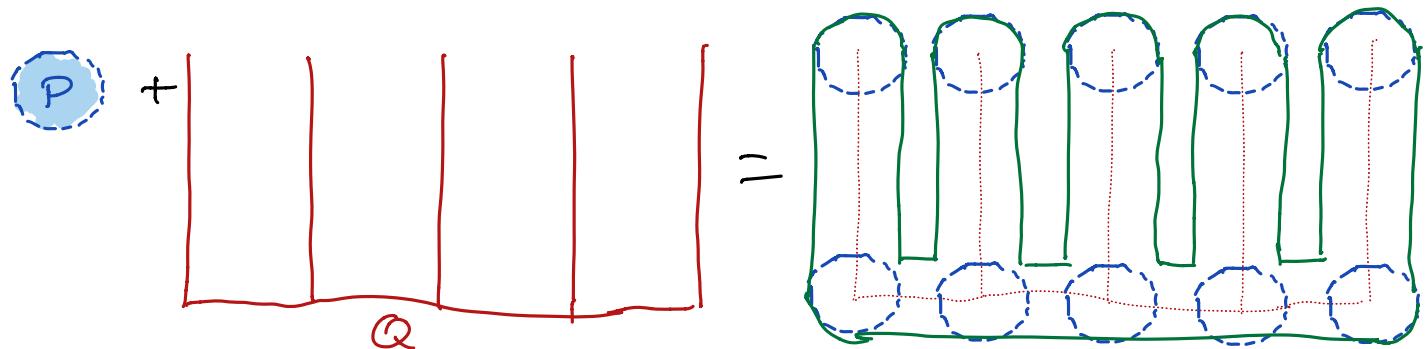
Decomposition: $(A \cup B) + (C \cup D)$

$= (A+C) \cup (A+D) \cup (B+C) \cup (B+D)$



$$\rightarrow P+Q = \bigcup_{i=1}^{m-2} \bigcup_{j=1}^{n-2} (P_i + Q_j)$$

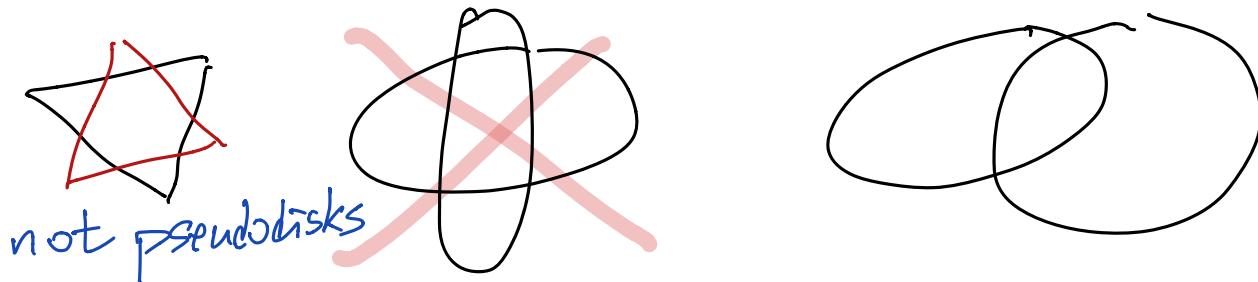
$O(mn)$ hexagons



IF P is convex but Q is not
 $P+Q$ can have $\Theta(mn)$

P, Q, Q' are all convex

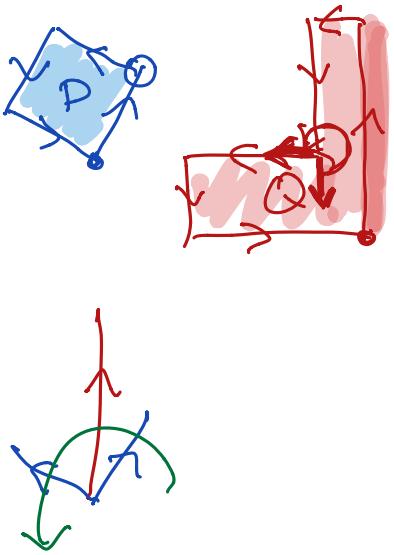
$P+Q$ and $P+Q'$ are pseudodisks



Union of pseudodisks with total of N vertices
 has complexity $O(N)$

② Convolutions! [Guibas Ramshaw Stoltz]

Generalize merging convex polygon idea
 to nonconvex polygons



$P * Q \longrightarrow$ collection of dir segments

- $P_i + (q_j \rightarrow q_{j+1})$
iff $P_{i-1} \rightarrow P_i$
 $q_j \rightarrow q_{j+1}$
 $P_i \rightarrow P_{i+1}$ are ccw order
- $(P_i \rightarrow P_{i+1}) + q_j$
iff --- ccw

IF P or Q convex, $P * Q$ is polygon
not simple

$P + Q =$ all points with positive winding #

) IF both $P + Q$ are nonconvex

$P * Q$ is directed graph

segments edges

Every component is Eulerian

