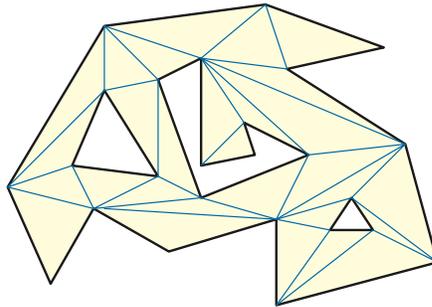


🌀 Homework 2 🌀

Due Monday, March 1, 2020 at 8pm

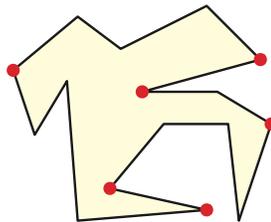
1. A *polygon with holes* is a connected region of the plane whose boundary consists of at least two disjoint simple polygons. The boundary of a polygon with holes consists of an outer polygon P_0 and one or more disjoint inner polygons P_1, P_2, \dots, P_h called *holes*; each hole lies in the interior of P_0 , but not in the interior of any other hole.

A *frugal triangulation* of a polygon with holes PH is a partition of PH into triangles, meeting edge-to-edge, with no additional vertices.



A frugal triangulation of a polygon with three holes.

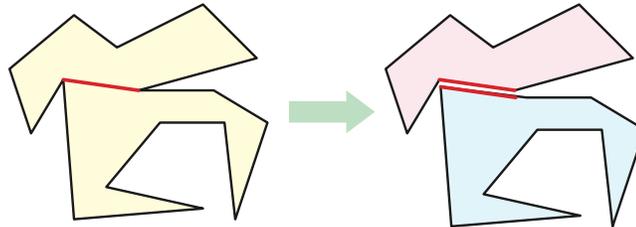
- (a) Let PH be a polygon with h holes, with a total of n vertices. How many triangles does every frugal triangulation of PH have, as a function of n and h ? Prove your answer is correct.
 - (b) Sketch an algorithm to compute a frugal triangulation of PH in $O(n \log n)$ time. You only need to describe any necessary changes from the algorithm described in class (and its analysis) for triangulating simple polygons. (Your algorithm will prove that every polygon with holes has a frugal triangulation!)
2. A *turn vertex* in a polygon is a vertex p_i whose neighbors p_{i-1} and p_{i+1} are both on the same side of the vertical line through p_i .



A polygon with six turn vertices (marked in red).

- (a) Describe an algorithm to sort the vertices of a given simple polygon from left to right in $O(n \log t)$ time, where n is the number of vertices and t is the number of turn vertices.

- (b) Sketch an algorithm to triangulate a simple n -gon with t turn vertices in $O(n \log t)$ time. You only need to describe any necessary changes from the algorithm described in class (and its analysis) for triangulating simple polygons.
3. An interior diagonal of a simple polygon P with $n \geq 4$ vertices is a *balanced separator* if it subdivides P into two smaller polygons, each with at least $\lceil n/3 \rceil + 1$ vertices. Describe and analyze an algorithm to find a balanced separator in a given simple polygon P (with at least four vertices). [Hint: Prove that a balanced separator always exists!]

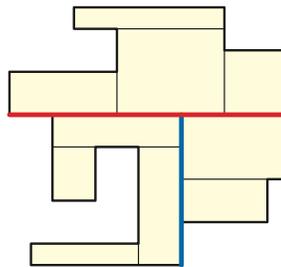


The red diagonal is a balanced separator of this polygon.

The problem originally asked for a diagonal with $\lceil n/3 \rceil - 1$ vertices on each side. Any bound of the form $n/3 \pm O(1)$ is worth 9 points out of 10.

4. A simple polygon is *orthogonal* if its edges alternate between horizontal and vertical. An orthogonal polygon is *generic* if no pair of edges lies on a common horizontal or vertical line. Let P be an arbitrary generic simple orthogonal polygon with n vertices.

A *rectangulation* R of P is a partition of the interior of P into axis-aligned rectangles. (These rectangles do not necessarily meet edge-to-edge.) A *bar* in a rectangulation R is a maximal connected union of collinear edges in R ; every edge of R is contained in exactly one bar. A rectangulation R of P is *proper* if (1) no vertex has degree 4 and (2) every bar in R contains an edge of the polygon P .



A proper rectangulation of a generic orthogonal polygon; the red and blue segments are bars.

- (a) Prove that P has exactly $n/2 - 2$ reflex vertices.
- (b) Prove that every proper rectangulation of P has exactly $n/2 - 1$ rectangles.
- (c) Describe an algorithm to construct a proper rectangulation of P . (In particular, this algorithm proves that a proper rectangulation exists!)
- (d) Prove or disprove: In every proper rectangulation R of P , every rectangle in R touches the boundary of P .

5. Prove that there are constants $0 < \alpha < 1$ and $\Delta > 1$ with the following property: In any planar map with n vertices, there is an independent set of size αn vertices, each with degree at most Δ . How small can you make the ratio Δ/α ? How quickly can you find such an independent set?