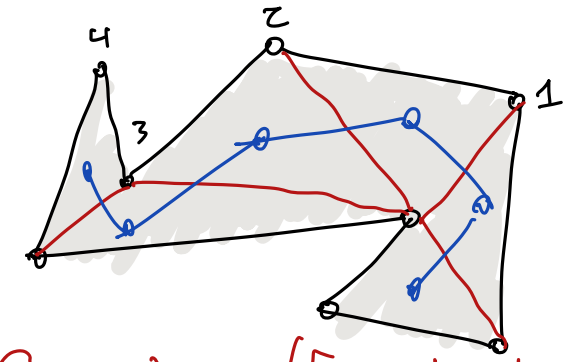
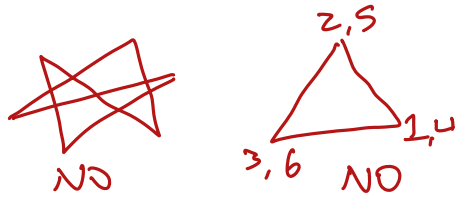
Polygon Triangulation

Input: Simple polygon P in plane



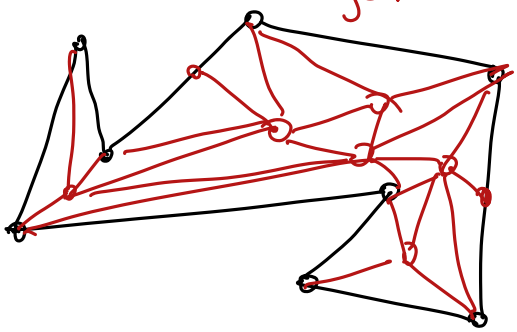
Specified as sequence of vertices in cyclic order



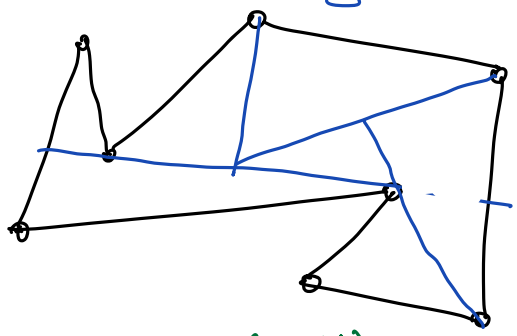
Compute a (frugal, interior) triangulation of P

Jordan curve theorem (1906)

Not frugal

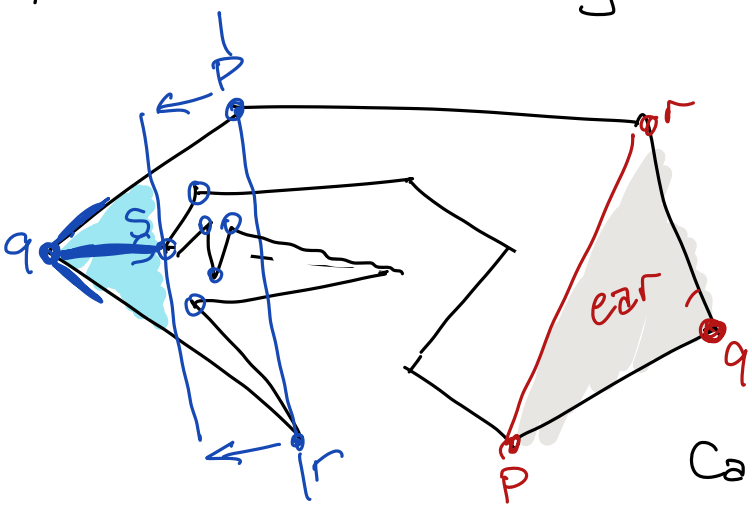


Not triangulation



Lennes (1911)
Dehn (1899) — unpublished

Find one interior diagonal



Lemma: Every simple polygon with >3 vertices has an interior diagonal

Proof:

Pick an extreme vertex q (min x, max x)

$p, r \leftarrow$ pred, succ of q

Case 1: pr is interior diagonal ✓

case 2: pr is not a (n interior) diagonal

P has vertices inside Δpqr

connect q to one of these vertices

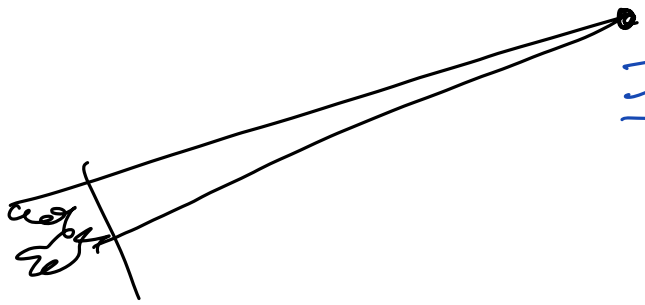
s = interior vertex furthest from $\overleftrightarrow{pr}$

qs is int. diagonal!

Algo: we can find a diagonal in $O(n)$ time

$\Rightarrow$ we can compute a triangulation in $O(n^2)$ time

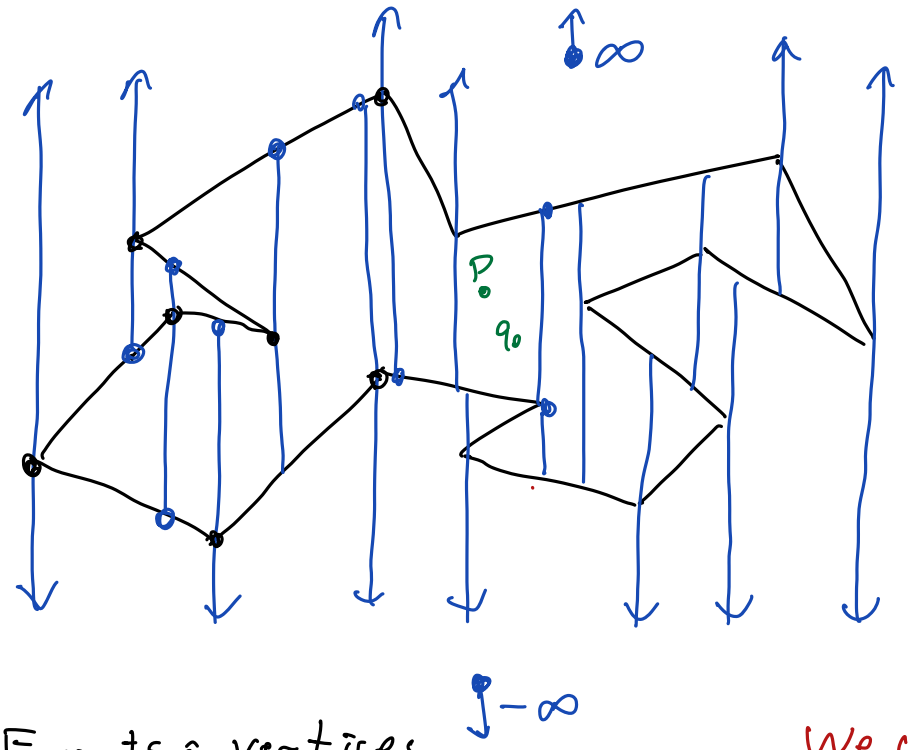
$$T(n) = T(k) + T(n+2-k) + O(n)$$
$$= O(n^2) \text{ worst case}$$



Interesting problem:

Find a diagonal splits vertices $n/3 + 2n/3$ or better.

$O(n \log n)$	today	$\leftarrow$ sweep line
$O(n \log^* n)$	later	
$O(n)$	Chazelle '90	Amato Goodrich Ramos 05?



Trapezoidal Decomposition

Shoot rays $\uparrow$ and $\downarrow$ from every vertex stop at first edge

PSLG

Events = vertices

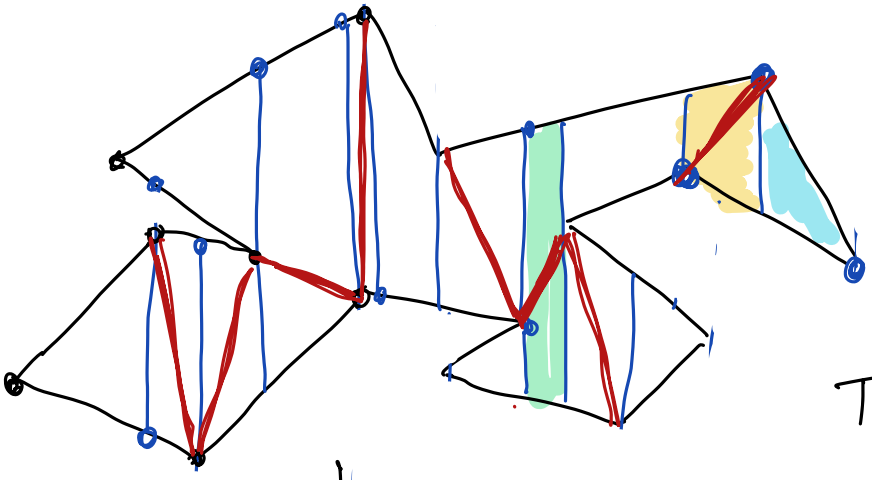
At each event in $\rightarrow$ order
find succ + pred
build walls up and down
add $O(1)$ verts and edges to trap-decomp graph.

We can build trap. decomp. in $O(n \log n)$ time using a sweep algo.

Define $p \sim q$ ("equivalent") if same edges above and below.

Trapezoid = equivalence class!

How to triangulate



① build trap decomp

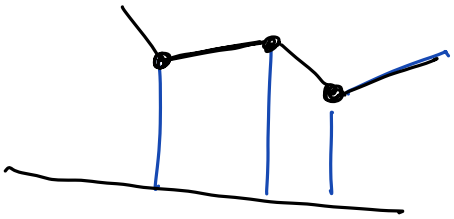
② discard exterior walls

Every interior trap has
a vertex on L wall
vertex on R wall

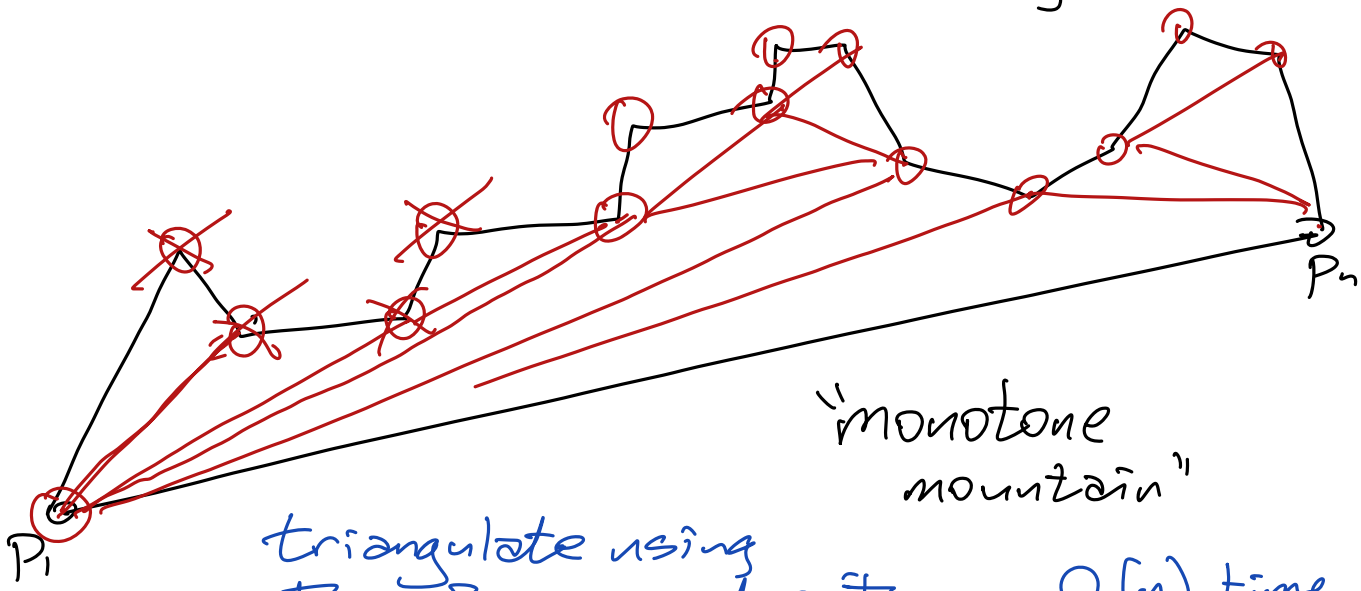
Trapezoid is <u>boring</u> unless
both on floor
or both on ceiling

Every boring trap contains a diagonal!

Suppose <u>no</u> boring trapezoids



Linear sequence of interesting traps
wlog all with defining pts
on the ceiling



"monotone mountain"

triangulate using the 3-penny algorithm $O(n)$ time