Planar straight-line graphs (PSLG) — geometric

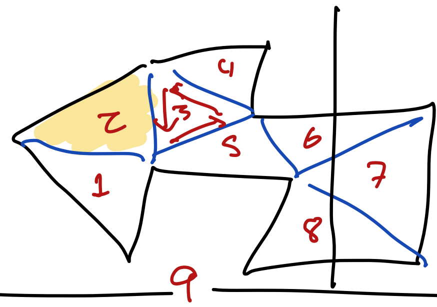
vertices = distinct points in $\mathbb{R}^2$

edges = interior-disjoint segments

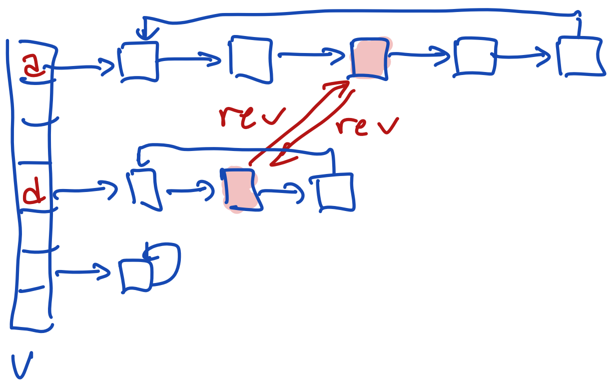
faces = regions/components of $\mathbb{R}^2 \setminus (V \cup E)$

Abstract graphs — combinatorial

Planar embeddings — topological

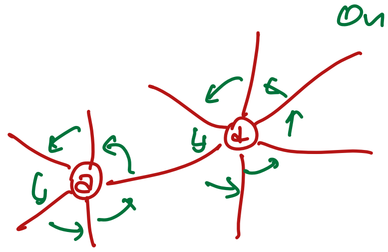


Standard data structure for multi-graphs:



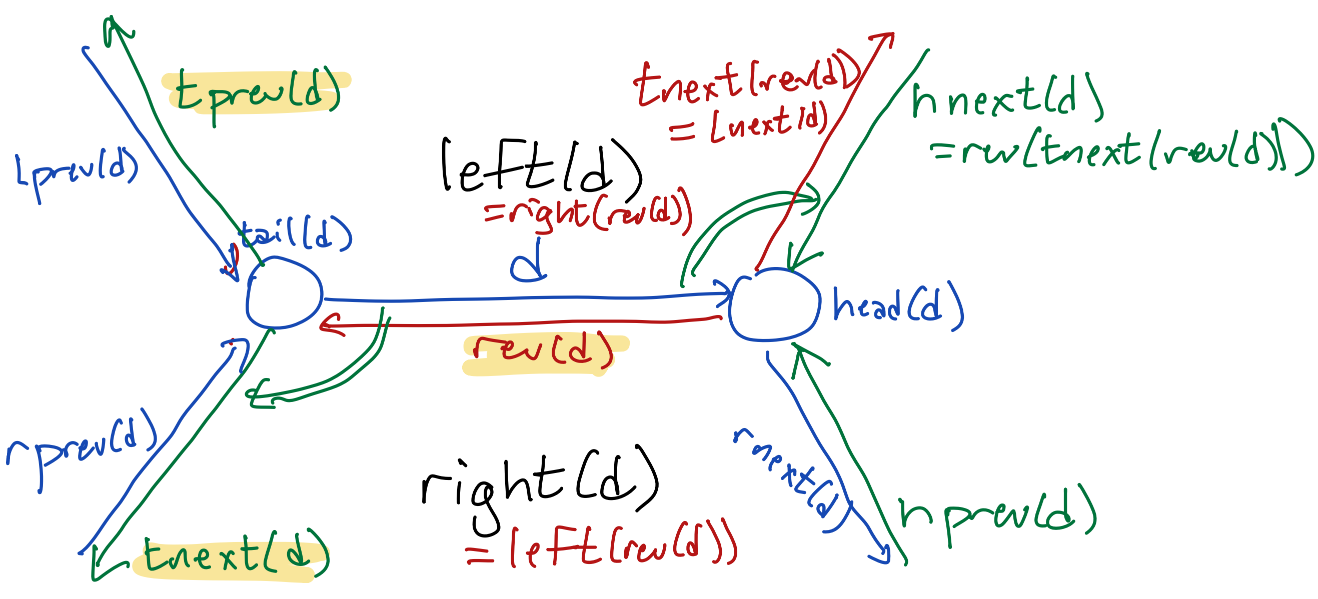
~~Adjacency~~ Incidence list

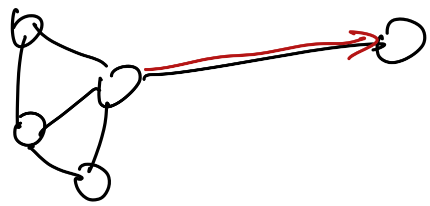
every vertex stores set of ~~adjacent vertices~~ ~~incident edges~~ outgoing darts

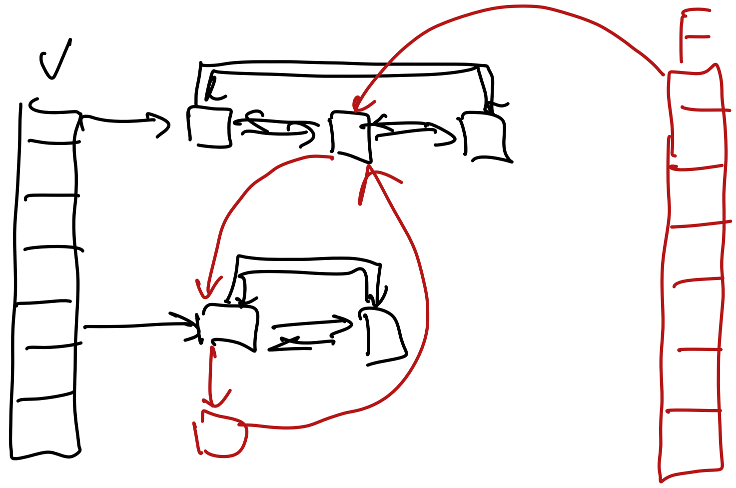


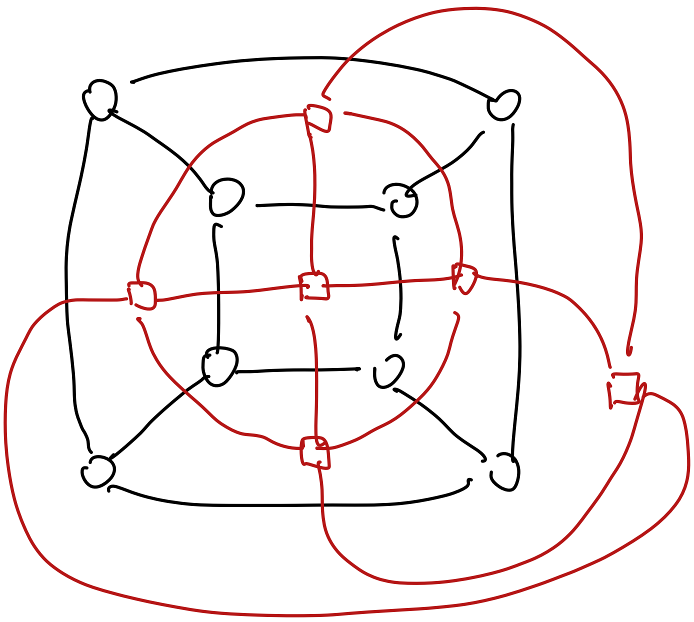
Every undirected edge is a pair of darts / half-edges

(Planar) embedding: incidence list stores circular sequence of darts out of each vertex





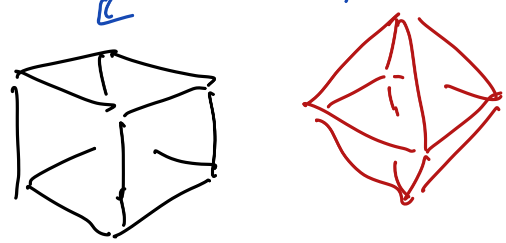


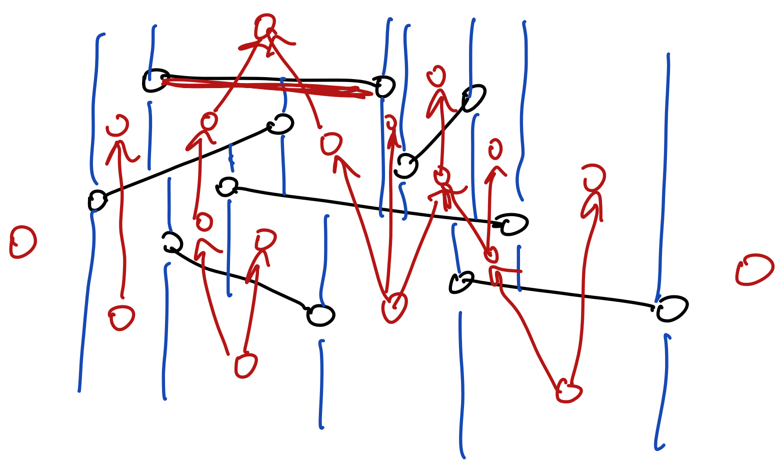


planar s.l. graph G

dual graph G^*

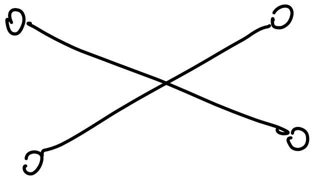
polarity duality





Building PSLG

Doubly-Connected Edge List (DCEL)

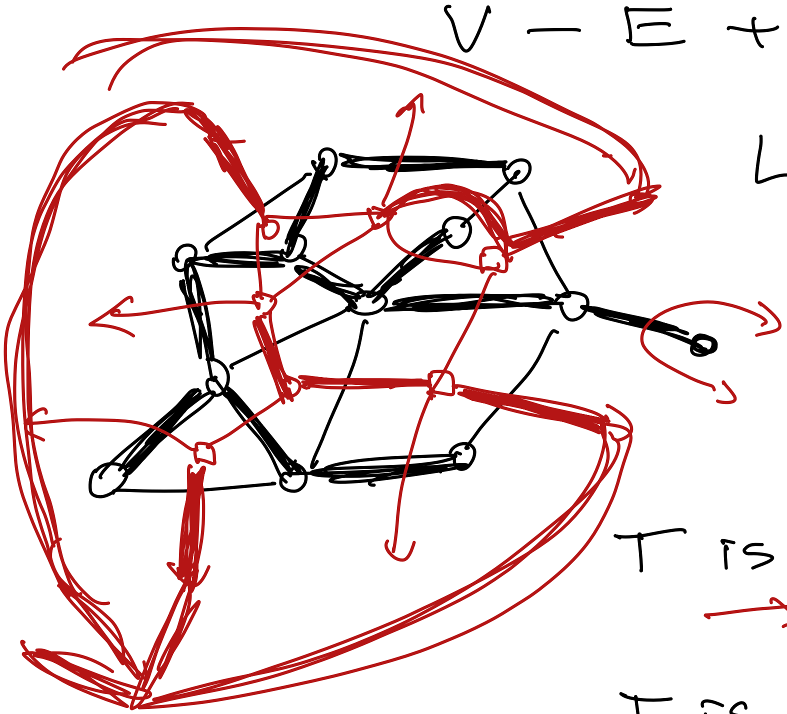


Things that look like they should take $O(1)$ time usually do.

Euler's formula

For any connected planar embedding

$$V - E + F = 2$$



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$\boxed{\text{Tree-cotree decomposition}}$

Let T be any spanning tree of G

$C = E \setminus T$

C^* = corresponding edges in G^*

T is connected

$\longrightarrow$ C^* is acyclic

T is acyclic

$\longrightarrow$ C^* is connected

$\Rightarrow$ C^* is a spanning tree of G^*!

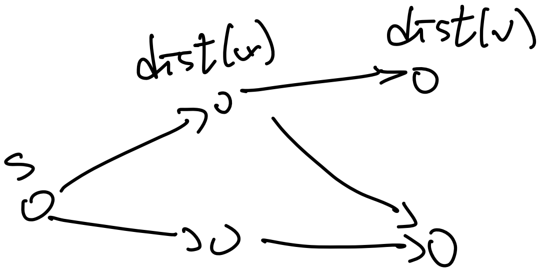
$$E = E(T) + E(C^*)$$
$$= V-1 + F-1 \qquad \checkmark$$

If G is simple

Every face has ≥ 3 edges

$\downarrow$

$$E \leq 3V-6 \qquad F \leq 2V-4$$



slack(u→v) = dist(u) + w(u→v) − dist(u)

slack(u→v) ≥ 0 if dists are correct

in dual = residual capacity