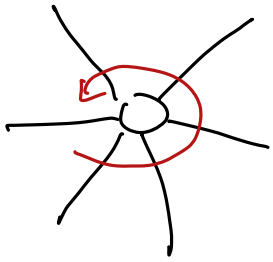
HW1 solutions out
HW2 out → due next Thu

Midterm due following Tuesday 8pm

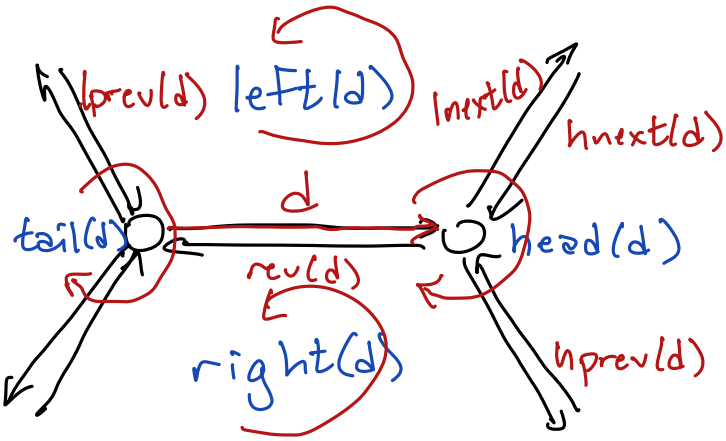
Last time: DCEL – data structure for

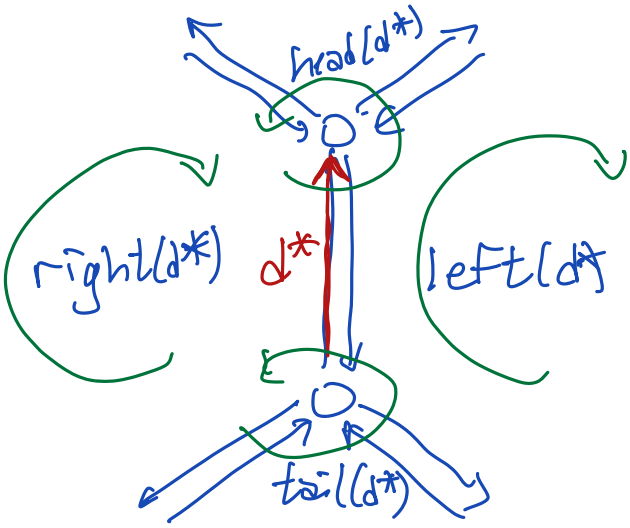


planar iff $V-E+F=2$

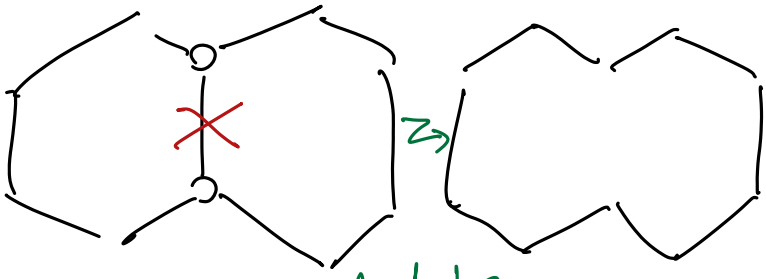
- ~~planar~~ maps
- ~~plane~~ graphs
- ~~planar~~ embeddings

planar straight-line graphs

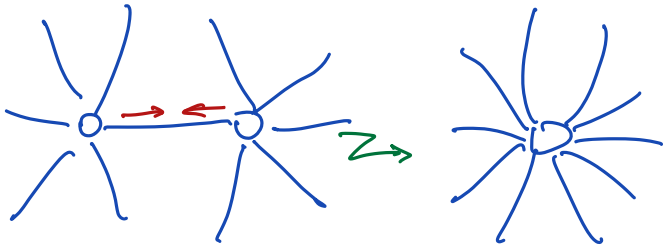




$$\text{left}(d)^* = \text{head}(d^*)$$
$$\text{right}(d)^* = \text{tail}(d^*)$$
$$\text{head}(d)^* = \text{left}(d^*)$$
$$\text{tail}(d)^* = \text{right}(d^*)$$



delete



contract

Euler's Formula: connected ~~simple~~ plane graph

$$V - E + F = 2$$

Base case: 0 $V=1$ $E=0$ $F=1$

Otherwise: $E \geq 1$

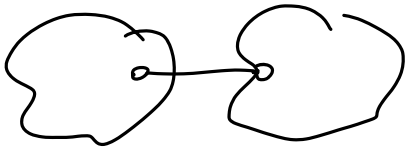
Pick any edge e

All deleted edges → dual spanning tree

Case 1: e is not a bridge

→ delete e

$E--$ $F--$ ✓

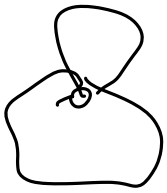


All contracted edges → spanning tree!

Case 2: e is not a loop

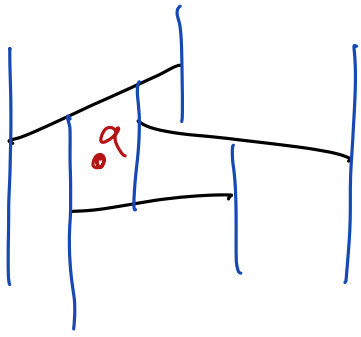
→ contract e

$E--$ $V--$ ✓



Trapezoidal decompositions in $O(n \log n)$ time ✓

How do we support point location?

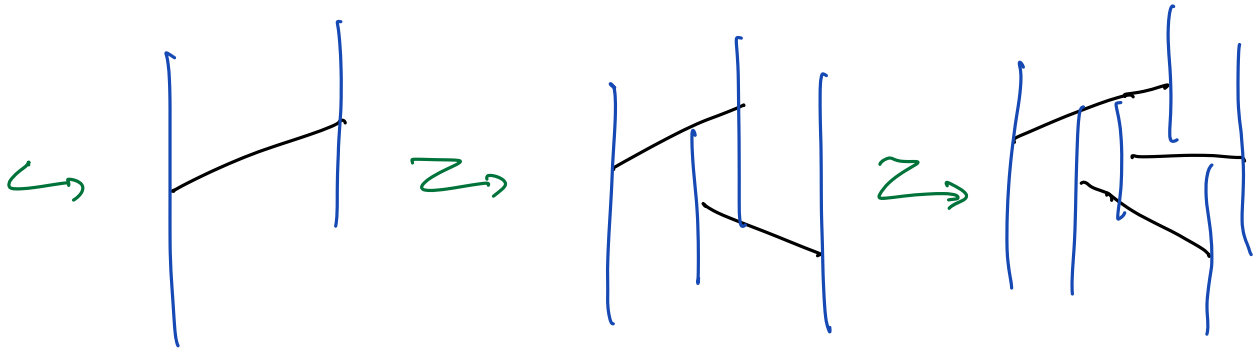


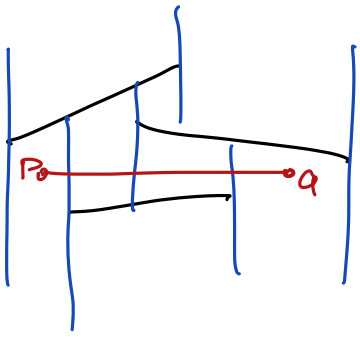
Don't sweep

Randomized Incremental Construction

Insert segments in random order

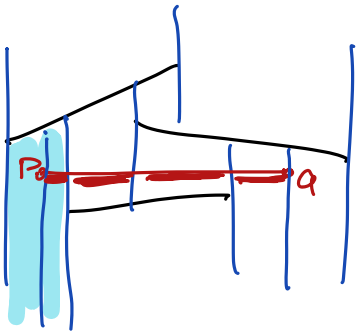
update decomp after each insertion.



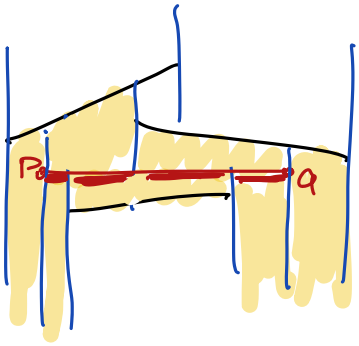


Inserting a new segment pq into an existing decomposition

- Locate p, split its trap — $O(1)$
- Walk along pq splitting traps — $O(1)$ each
- Reach q, split its trap — $O(1)$
- Merges — $O(1)$ each

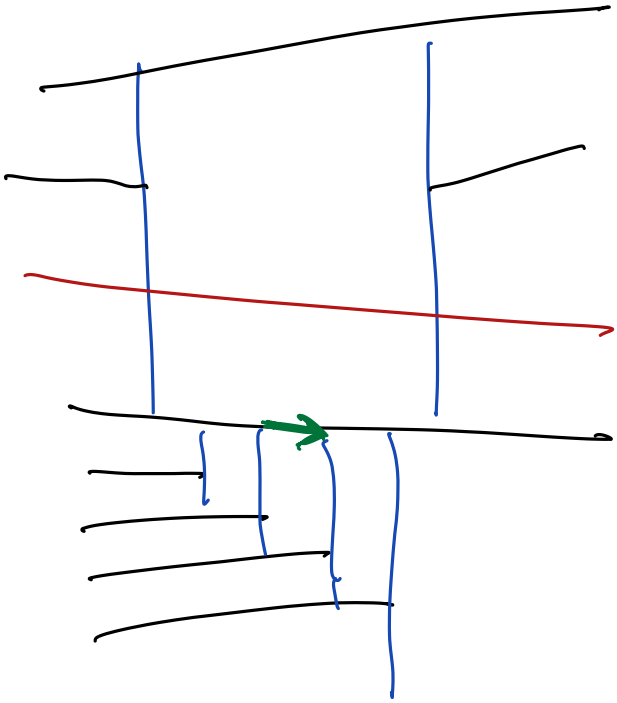


Define $\deg(pq)$ = # traps in new decomposition that touch pq



$\Rightarrow O(\deg(pq))$ + pt. loc. time in worst case

$= O(n)$



Updating a DCEL is inefficient (unless we are clever)

So don't do that.

Lightweight data structure

Store set of trapezoids

Each trap stores
- ≤ 4 segments defining it
- ≤ 4 neighboring traps thru vertical walls

Backward analysis — imagine running randomized incremental algo backward

randomized decremental algorithm

Choose a random segment
Delete it from trap decomp ← Expected time $O(E[\deg(s)]) = O(1)$
Recurse

Total expected time $= O(n)$

What is $E[\deg(s)]$ where s is chosen randomly?

$$\| $$

$$\frac{1}{n} \cdot \sum_{i=1}^{n} \deg(s_i) = \frac{1}{n} \cdot \sum_{i=1}^{n} \sum_{j=1}^{N} [s_i \text{ touches } \Delta_j]$$

trapezoid

$$= \frac{1}{n} \cdot \sum_{j=1}^{N} \sum_{i=1}^{n} [s_i \text{ touches } \Delta_j]$$

trapezoid

Euler $\Rightarrow N = O(n)$

$$\leq \frac{1}{n} 4N = O(1)!$$