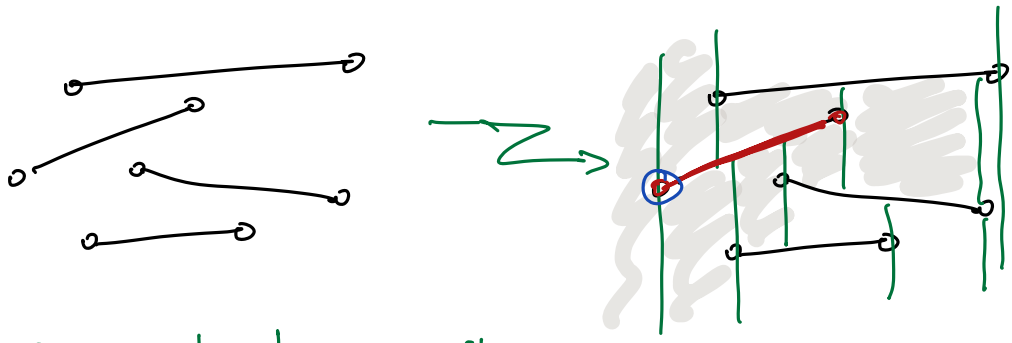
COVID policy changes — see recent Massmail

Randomized incremental construction



$T_0 \leftarrow$ empty decomposition

for $i \leftarrow 1$ to n in random order

insert s_i into $T_{i-1} \longrightarrow T_i$

Last time: cost to insert s_i

$= \text{time to locate } p_i + O(\deg_i(s_i))$

traps in T_i that touch s_i

$E[\text{cost insert } s_i] = E[\text{time to locate } p_i] + O(E(\deg_i(s_i)))$

Backward analysis

$\boxed{s_i \text{ is a random segment in } T_i}$

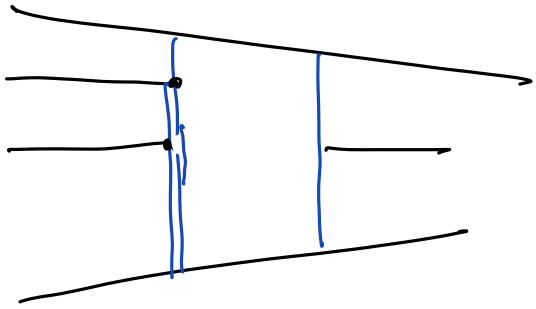
$\sum_{s \in T_i} \deg(s) = O(i)$ by Euler's formula

so $E[\deg_i(s_i)] = O(1)$

$E[\text{time to locate } p_i] + O(1)$

$\Rightarrow$ Total expected time $= O(n)$

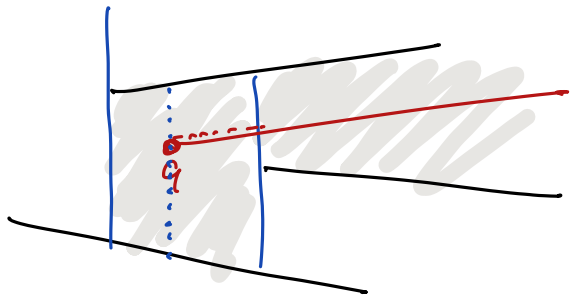
$\leftarrow E[\text{time for all } n \text{ point locations}]$



NOW?

① Conflict lists

Each trapezoid maintains the set of endpts inside it $K(\Delta)$
Each point maintains trap that contains it



When we split any trapezoid Δ we traverse $K(\Delta)$ by brute force and reassign every point

$$\boxed{O(|K(\Delta)|) \text{ time}}$$

$$\text{Total time} = \sum_{\Delta \in T_{i-1} \setminus T_i} O(|K(\Delta)|) \qquad \text{deleted traps}$$

$$= \sum_{\Delta \in T_i \setminus T_{i-1}} O(|K(\Delta)|) \qquad \text{created traps}$$

$$= \sum_{\substack{\Delta \in T_i \\ \text{touch } s_i}} O(|K(\Delta)|)$$

$$= O\left(E[\#\text{ endpoints } p_j \text{ whose trap changes}]\right)$$

$$= O\left(\sum_{j>i} \Pr[\underbrace{\text{trap containing } p_j \text{ changes}}]\right)$$

$$X = \sum_j X_j \qquad X_j = 0 \text{ or } 1$$

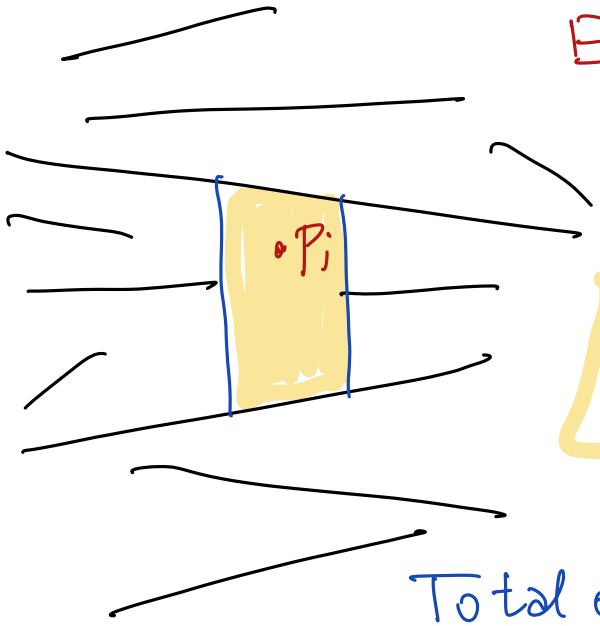
$$E[X] = E\left[\sum_j X_j\right] = \sum_j E[X_j] = \sum_j \Pr[X_j = 1]$$

$$E[X_j] = 0 \cdot \Pr[X_j = 0] + 1 \cdot \Pr[X_j = 1] = \Pr[X_j = 1]$$

Total over all iterations

$$\text{Total exp. time} = O\left(\sum_{i=1}^{n} \sum_{j=i+1}^{n} \Pr[\Delta_i(p_j) \neq \Delta_{i-1}(p_j)]\right)$$

$$= O\left(\sum_{j=1}^{n} \underbrace{\sum_{i=1}^{j-1} \Pr\left[\Delta_i(p_j) \neq \Delta_{i-1}(p_j)\right]}_{E[\#\text{ times trap containing } p_j \text{ changes}]}\right)$$



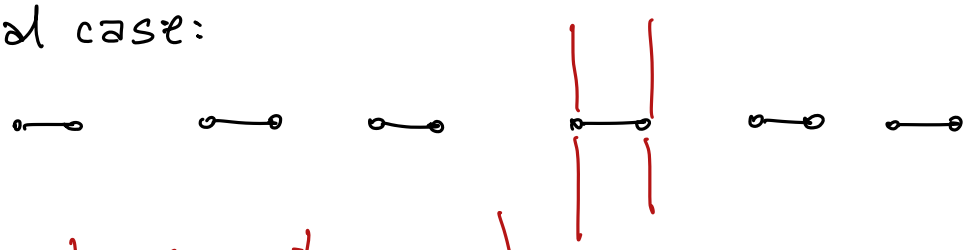
$$\boxed{\Pr\left[\Delta_i(p_j) \neq \Delta_{i-1}(p_j)\right] \leq \frac{4}{i}}$$

$$\text{Total exp. time} \leq O\left(\sum_{i=1}^{n} \sum_{j=i+1}^{n} \frac{4}{i}\right)$$

$$\leq O(n) \cdot \underbrace{\sum_{i=1}^{n} \frac{1}{i}}_{\text{Harmonic number } H_n} = O(n \log n)$$

$$\ln(n+1) \leq H_n \leq \ln n + 1$$

Special case:



Choose random segment

Partition

Recurse on both sides

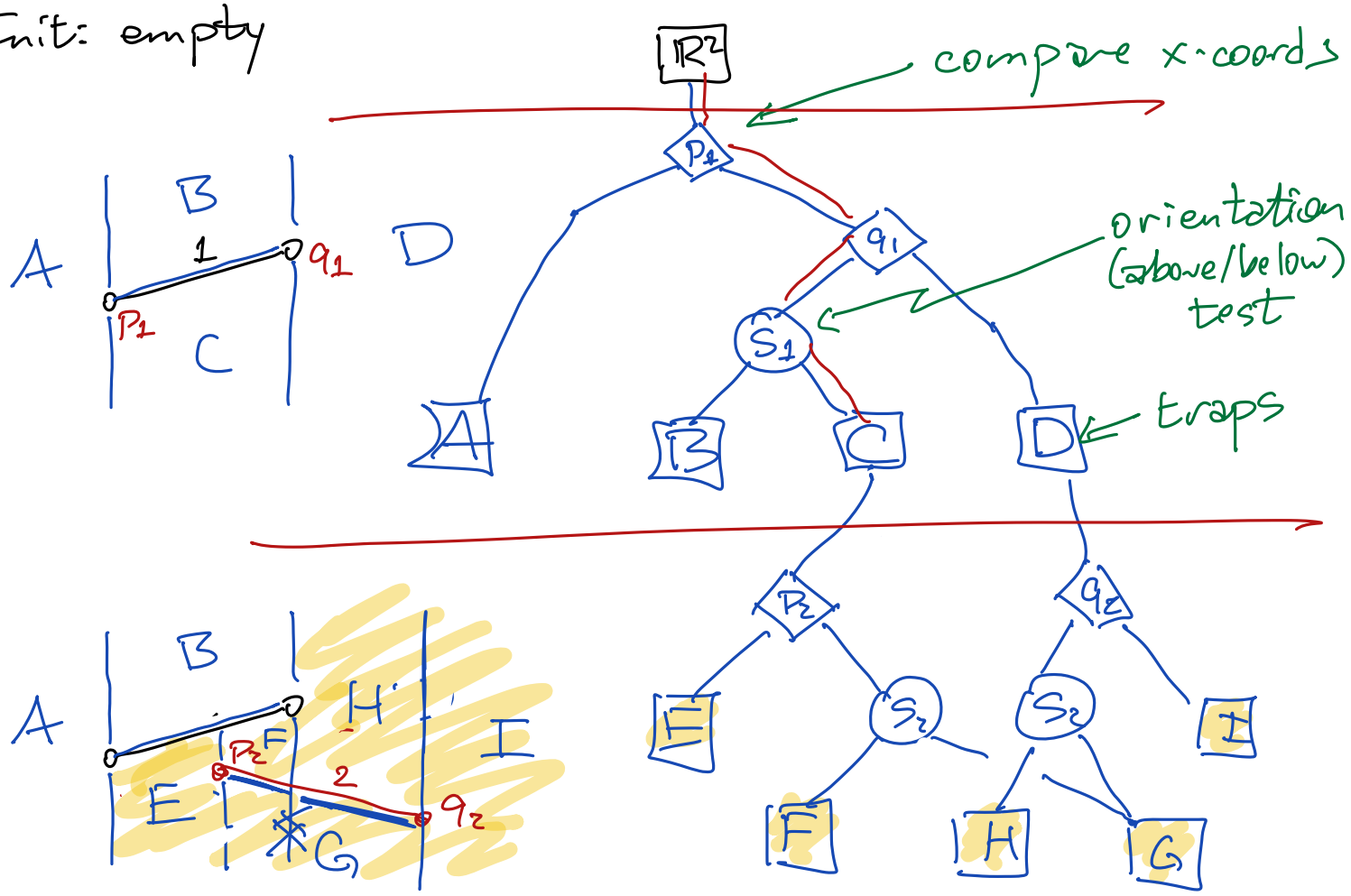
Randomized quicksort!

Conflict list = recursive subproblem

History dag = binary search tree (treap)

② History dag Build = pt. loc. data structure

Init: empty



$\mathbb{E}[\#\text{ nodes per level}] = O(1) \Rightarrow \mathbb{E}[\text{space}] = O(n)$

<u>depth</u> per level ≤ 3

Time to locate $q \leq 3 * \#$ levels where trap containing q changes.

$$\mathbb{E}[\text{Time to locate } q] \leq 3 \cdot \sum_{i=1}^{n} \Pr(\Delta_i(q) \neq \Delta_{i-1}(q))$$

$$\leq 3 \cdot \sum_{i=1}^{n} \frac{4}{i} = 12 H_n = O(\log n)$$