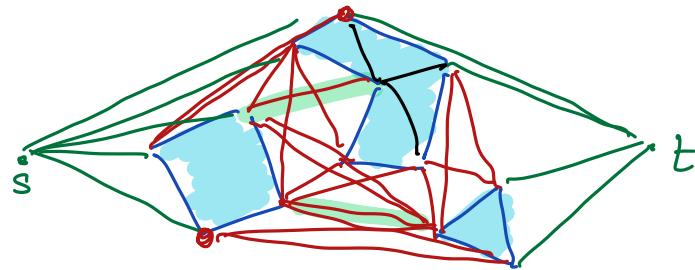


Talks next week  
No class following wed  
Final Exam due May 11

ICES Forms

## Visibility Graphs



Polygonal obstacles  $P_1 \dots P_k$

Vertices = vertices of obstacles

Edges = free segments between vertices

Shortest path from  $s$  to  $t$  is sequence of edges  
in  $\text{Vis}(P_1 \cup P_2 \cup \dots \cup P_k \cup s \cup t)$

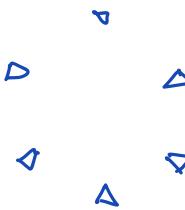
Many edges in Vis Gr (between convex vertices)  
lies on some shortest paths.

$\text{Vis}(P)$  is not planar

Best case:  $\Theta(n)$



Worst case:  $\Theta(n^2)$

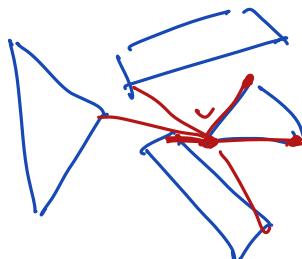


WLOG obstacles are  $\Delta$ s or segments

### Naive construction:

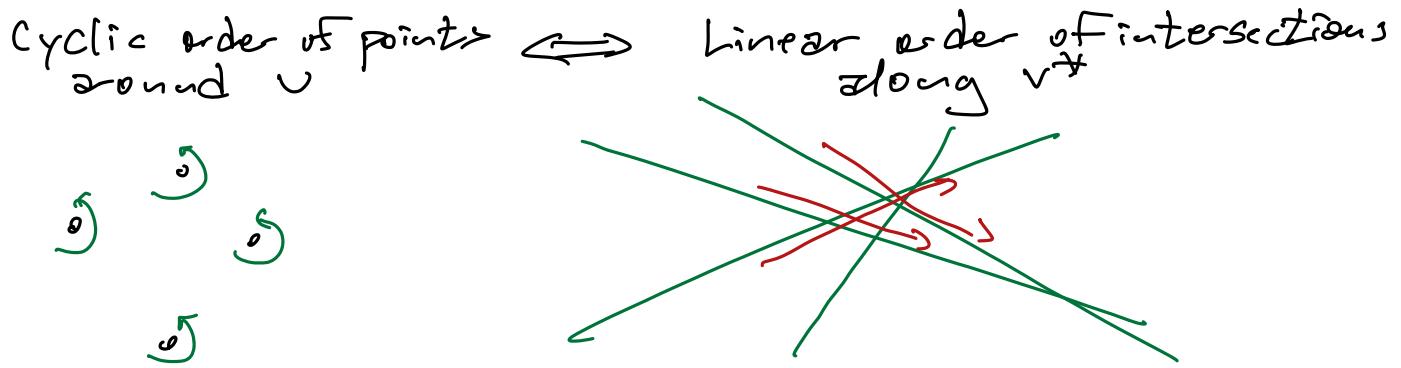
Brute force: For every pair of vertices check visibility,  
 $\Theta(n^3)$  time

For each vertex  $v$   
Find all vertices visible to  $v$   
via rotational sweep  $\rightarrow \Theta(n \log n)$  time per vertex  $v$

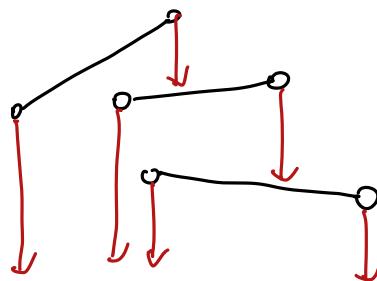


$\Rightarrow \Theta(n^2 \log n)$  time

# Duality



All rotations at once:



Every vertex  $v$  maintains (index of) next object below(v)

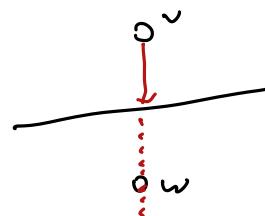
Rotate all rays, watch for changes in below(v)

Four events: when ray from some vertex  $v$  hits vertex  $w$

(1) Input:  $vw$  is input segment  
add it to  $G$ .

(2)  $\text{below}(v)$  crosses  $vw$

Do nothing



(3)  $\text{below}(v) = \text{below}(w)$

Add  $vw$  to  $G$

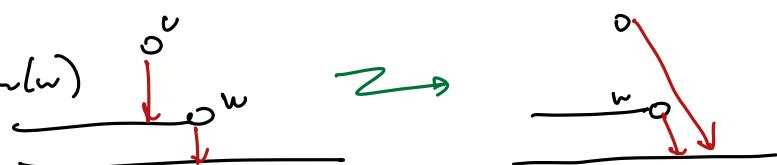
$\text{below}(v) \leftarrow \text{seg}(w)$



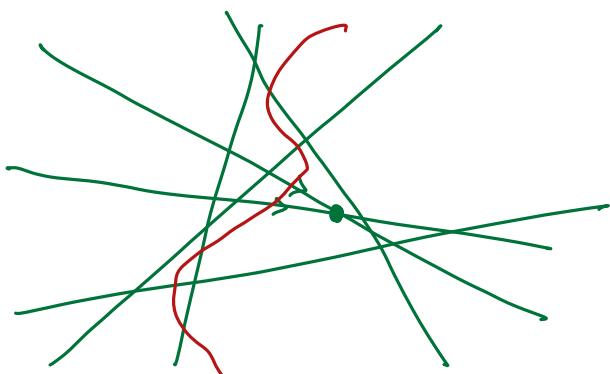
(4)  $w$  is endpoint of  $\text{below}(v)$

Add  $vw$  to  $G$

$\text{below}(v) \leftarrow \text{below}(w)$



$\Rightarrow O(n^2 \log n)$  time  
dominated by ordering events.



topological sweep

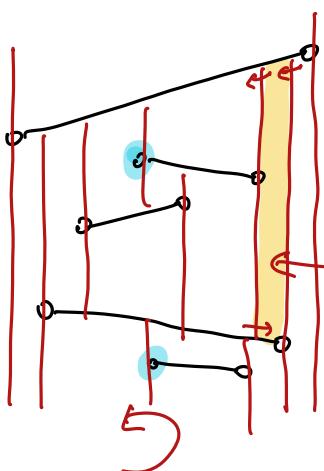
(with more data structures  
don't build graph first)

Events can be handled onto Forder

Direct edges in dual graph  
left to right

Process events in any  
topological order!

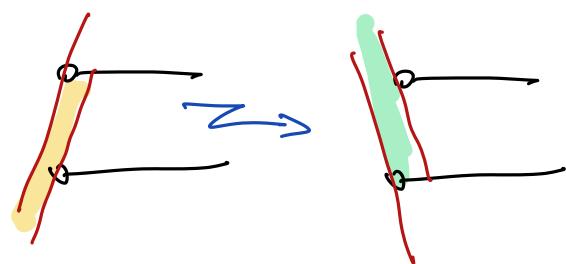
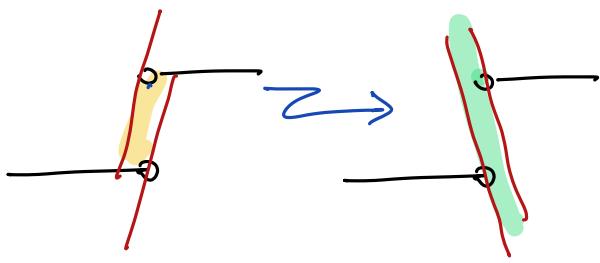
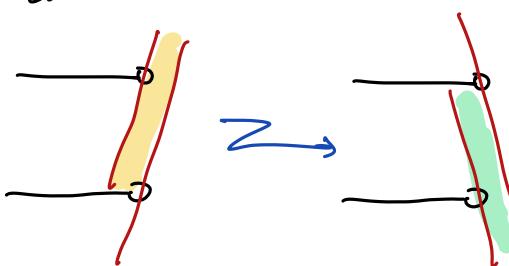
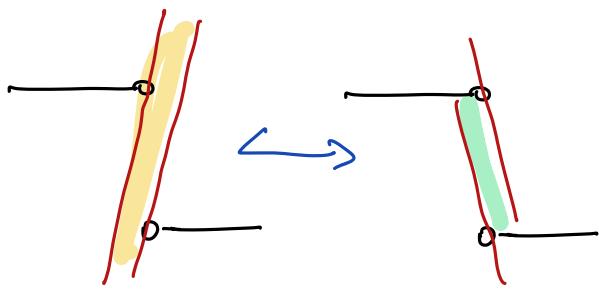
$\Rightarrow \underline{\mathcal{O}(n^2)}$  time!



Maintain trapezoidal decomposition  
as defining direction rotates.

This trap is about to collapse

We discover visibility edges when  
trapezoids collapse  
we don't notice other "events"  
where direction is parallel to  
line thru two vertices



We can still handle each event in  $\mathcal{O}(1)$  time

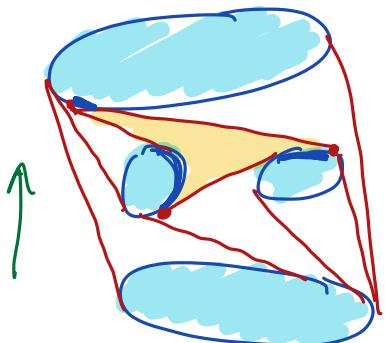
#events = #vis edges =  $K$

$\mathcal{O}(\log n + K \log n)$



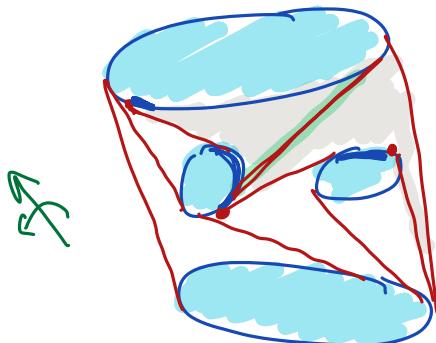
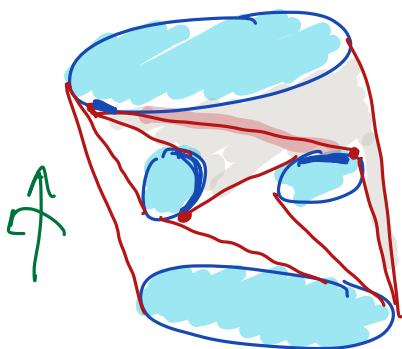
[Pocchiola + Vegter 94]

Combine local events and topological sweep



Obstacles are convex sets

Use "greedy pseudotriangulation"  
instead of top decomp.



Every edge of pseudotriangulation has a "Flip time"

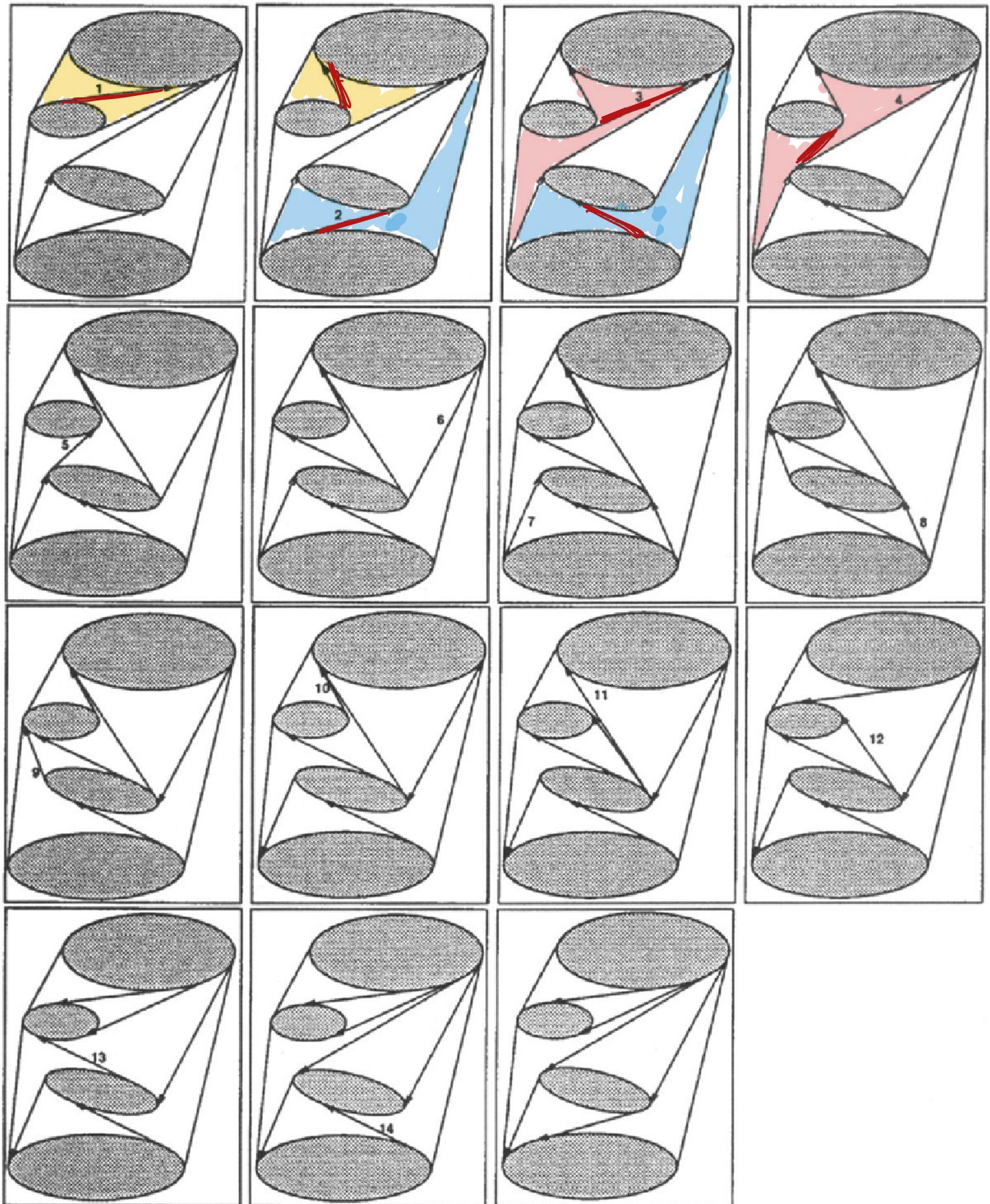
Keep flip times in a priority queue

Each flip  $\rightarrow O(1)$  time +  $O(1)$  PQ operations  
 $\rightarrow O(n \log n + K \log n)$  time

Don't need to handle flips in chron order

Local info tells us if an edge is safe to flip

$\Rightarrow O(n \log n + K)$  optimal



**Fig. 10.** The greedy flip algorithm. At each step the internal bitangent of minimal slope in the current pseudotriangulation is flipped.

