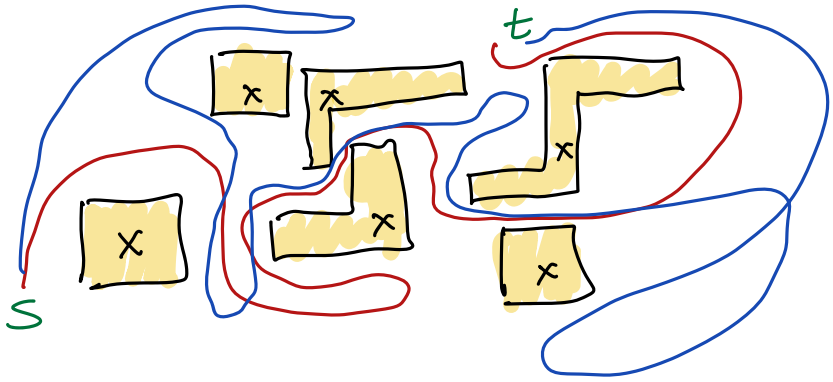


Is A homotopic to B?

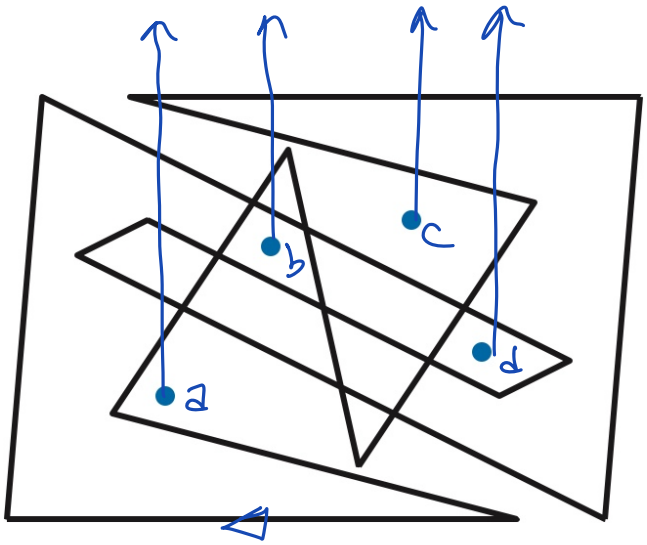
$\Updownarrow$

Is $A \cdot \overline{B}$ contractible?

$O(x+n)$ sort

$\downarrow$ $\downarrow$

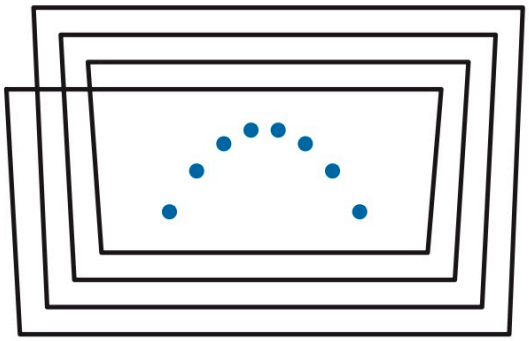
$O(nk + k \log k)$ time

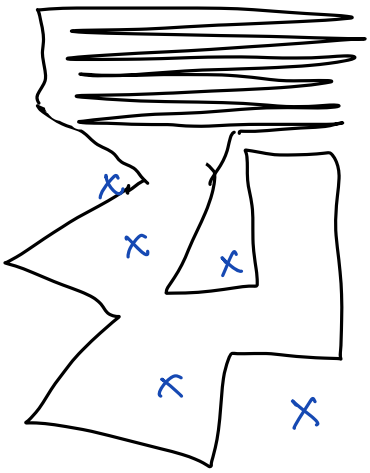


contractible!

$\cancel{\alpha\beta\alpha\gamma\alpha\beta\alpha\gamma\alpha\beta\alpha} = \varepsilon$

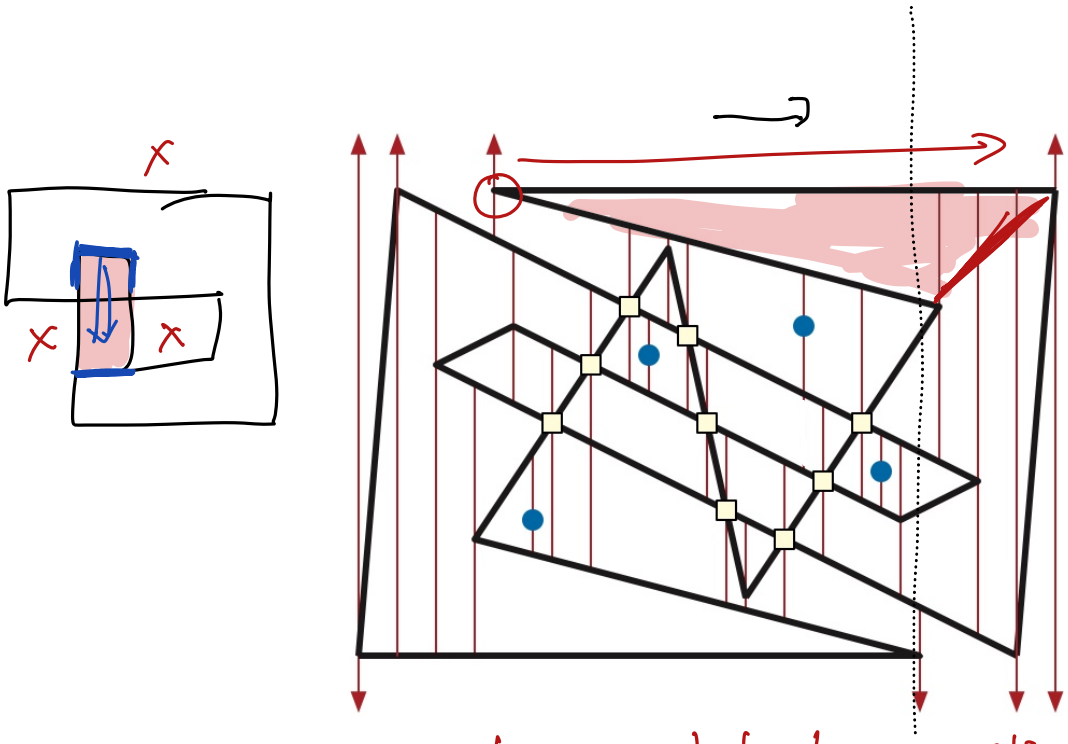
reduced crossing sequence has length $\Omega(nk)$





$O((n+k)\log n)$ with correct preprocessing

Special case: Few self-intersections



trapezoidal decomposition

Extend walls up + down from
- vertices of P
- obstacles O
- self-intersection points

Sweep-line [Bentley-Ottman]

Move vertical line $\longrightarrow$

Maintain sorted sequence of intersections (indices)

Maintain a priority queue of potential
future self-intersections
+ obstacles
+ vertices of P

Each event $\rightarrow$ $O(1)$ ins/del in intersection seq
balanced binary tree $\rightarrow O(\log n)$
+ $O(1)$ ins/del in event queue
$\rightarrow O(\log n)$

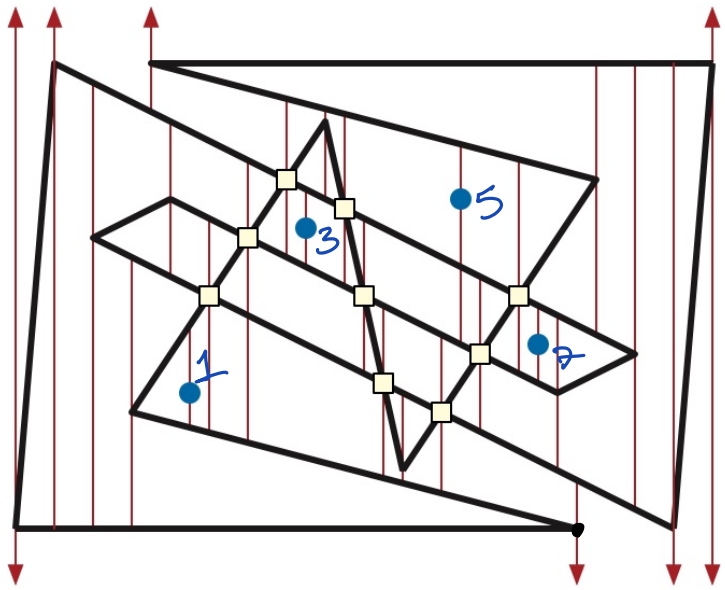
$\#\text{events} = n + k + s$

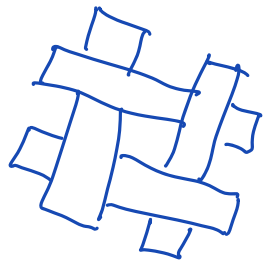
$\#$self-ints

$\Rightarrow O((n+k+s)\log n)$ time

vertical and horizontal ranking.

Goal:
Replace
edge → —
vertex → |

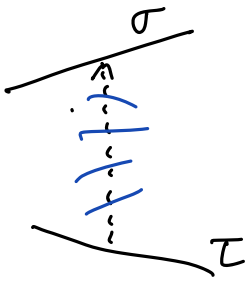


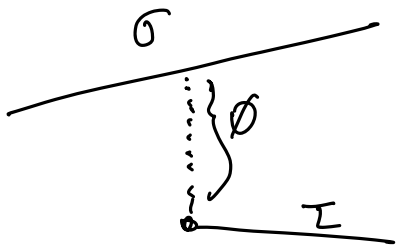


horizontal rank of <u>vertex</u> = 2 × # obstacles to left
<u>obstacle</u> = 2 × # obstacles to left + 1

Vertical ranking:

$\sigma \Uparrow \tau$
"σ is above τ"





$\sigma \uparrow \tau$
"directly above

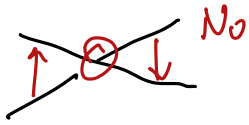
$\sigma \Uparrow \tau$ is the transitive closure of $\sigma \uparrow \tau$

For the sake of argument, suppose

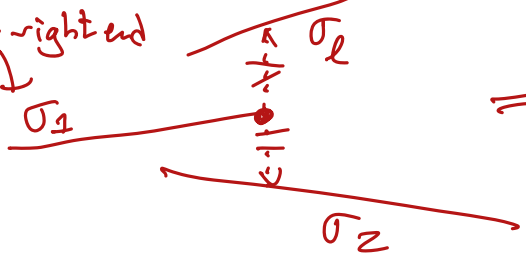
$$\sigma_1 \Uparrow \sigma_2 \Uparrow \sigma_3 \cdots \sigma_\ell \Uparrow \sigma_1$$

shortest such cycle.

① $\ell = 1$ ✗ $\ell = 2$



leftmost-right end



$\Longrightarrow \sigma_\ell \Uparrow \sigma_2$ contradicting

□

Define fine vertical ranks for edge fragments and obstacles.
from any topological order of the dag ↑

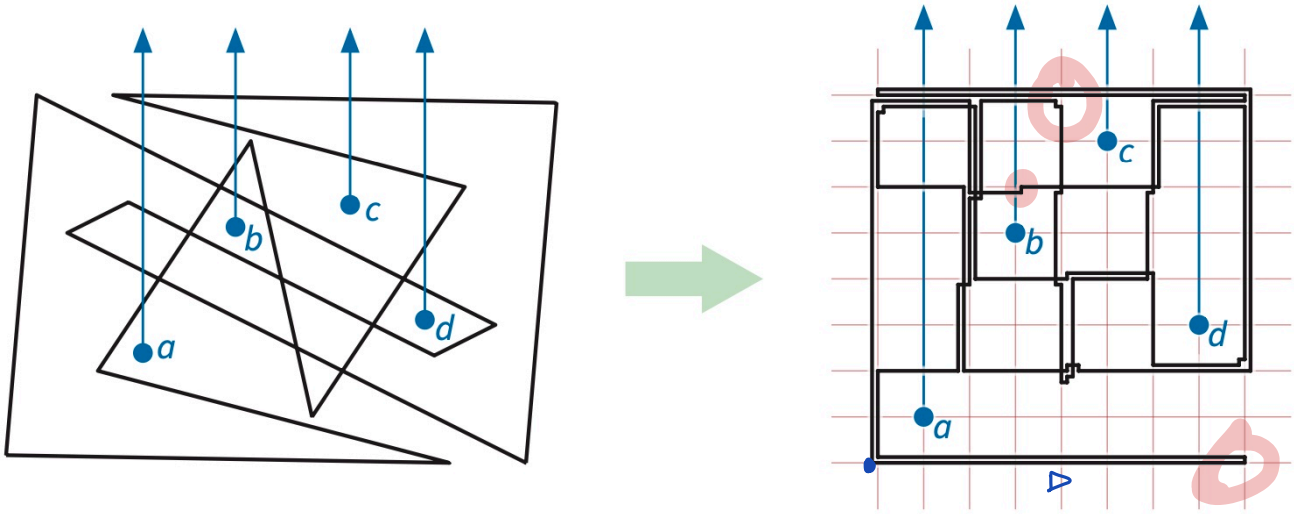
vert rank obstacle = 2 # obstacles with lower finevrank + 1
fragment = 2 # obst with lower fine v. rank

replace obstacle $(x,y) \longrightarrow$ (hrank, vrank)

vertex q (hrank(q), vrank(pq))
(hrank(q), vrank(qr))

edge pq (hrank(p), vrank(pq))
(hrank(q), vrank(pq))

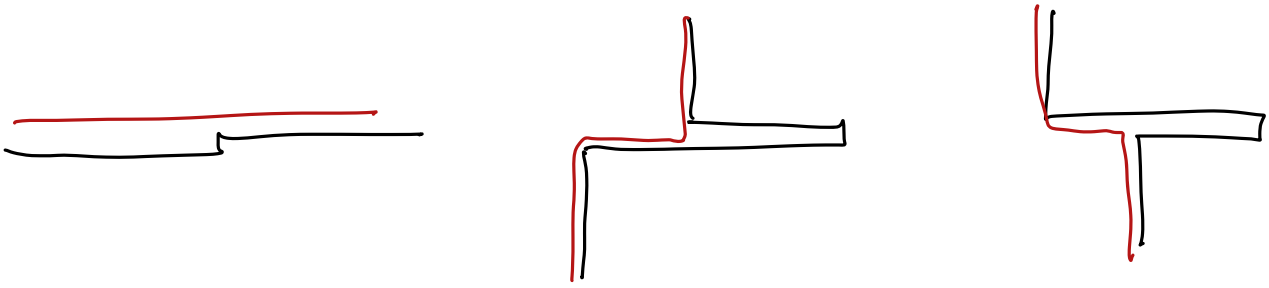
Represent $\overline{P}$ rectified polygon
by alternating x- and y-coordinates.



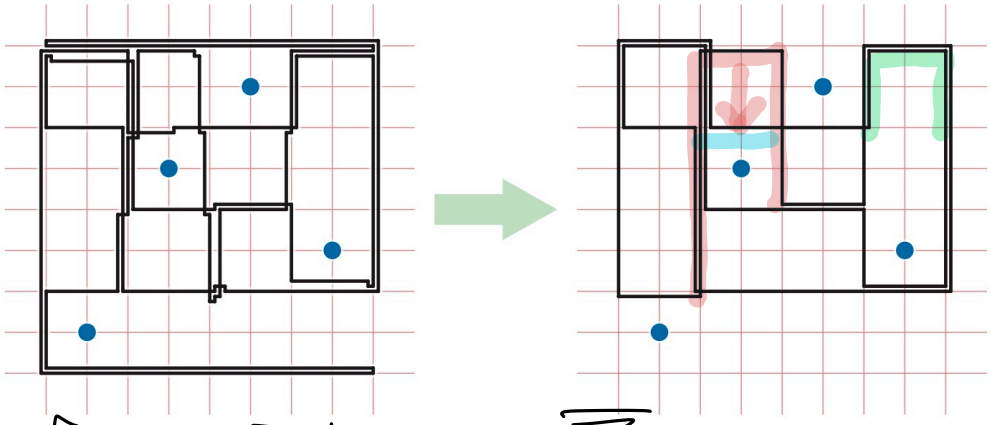
same crossing sequence

x y x y x y x

0 0 0 0 0 2 2 . . . .

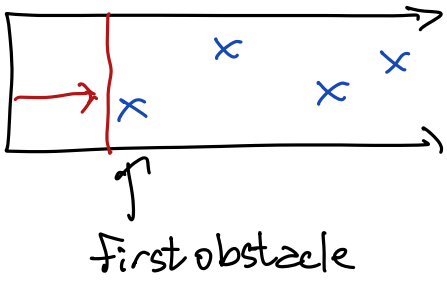


Remove zero-length edges



To simplify rectified polygon $\overline{P}$

<u>slide brackets</u>



Store k points $\overline{O}$

Answer queries:

Given a bracket (x, y, x)

How far can we slide it?

$O(\log k)$ time

$\#\text{slides} \leq O(n+s)$