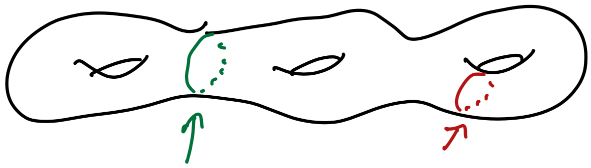
Shortest Interesting Cycles

Spreadsheet to register projects



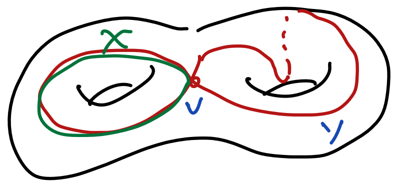
Given a surface map Σ, find shortest noncon/nonsep cycle.
positively weighted edges

closed walk

simple closed walk

Lemma: Shortest noncon/nonsep closed walk is a simple cycle.

Proof: Spose not.



W = shortest nontrivial closed walk
Spose W is not simple

$$W = X \cdot Y$$

X and Y shorter than W $\Rightarrow$ both trivial.

Non-contractible $\Rightarrow$ W is contractible ※

Non-separating $\Rightarrow$ $X = \partial A$ $\quad Y = \partial B$ $\quad W = \partial(A \oplus B)$

W is separating ※

Surface Mesh Simplification

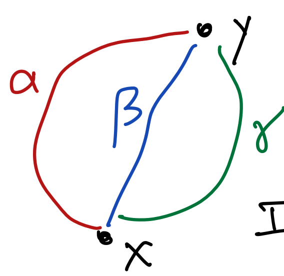
Triangulation 100M Δs $\longrightarrow$ 1000 Δs "same" geometry

Topological Data Analysis

reconstructed surfaces are noisy
topological noise is bottleneck to simplification

Thomassen $O(n^3)$

3-path condition



Three cycles:
α·rev(β)
β·rev(γ)
α·rev(γ)

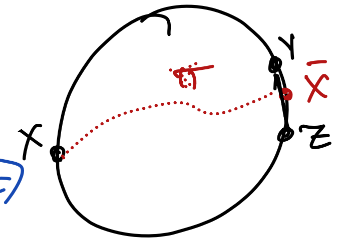
If two are trivial, so is third.

⇒ Let σ be shortest nontrivial cycle in Σ

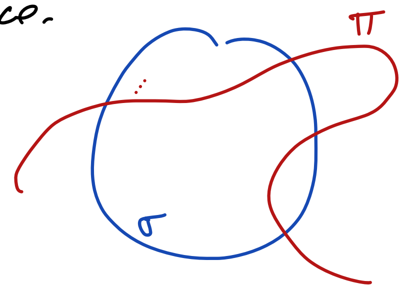
x any pt in σ $\quad$ $\bar{x}$ antipodal point in σ

Both paths from x to $\bar{x}$ in σ are shortest paths in Σ

(same exchange argument)



⇒ Any shortest path crosses σ at most once.



$O(n^3)$ time algorithm:

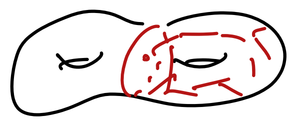
Let $\text{loop}(x, yz) = \text{path}(x \rightsquigarrow y) \cdot yz \cdot \text{path}(z \rightsquigarrow x)$ $\quad$ shortest!

There are $O(n^2)$ loops like this

Enumerate in $O(n^2 \log n)$ time $\quad$ Dijkstra $\times n$

Is $\text{loop}(x, yz)$ contractible? → $O(n)$ Dehn's algo.

separating? → WFS in Σ^* $\quad O(n)$

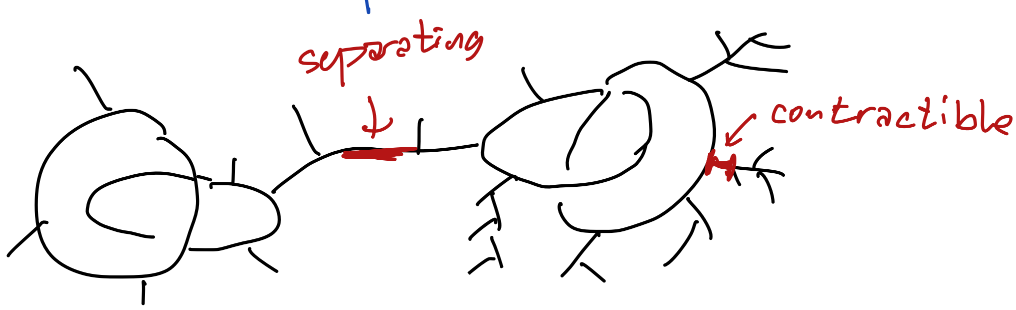


[E. Har-Peled]

Tree-cotree decomposition (T, L, C) $\quad$ shortest path tree @x

Dual cut graph $C^* \cup L^* = X$

Every $\text{loop}(x, yz)$ crosses X once at yz.



$\text{loop}(x, yz)$ is separating iff $(yz)^*$ is bridge of X

contractible iff (yz) is not in reduced(X)

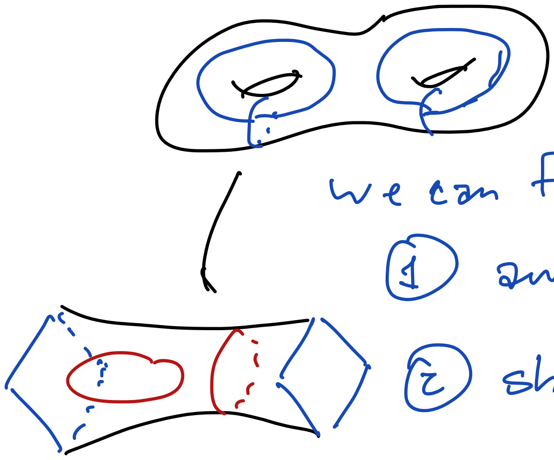
We can test all $\text{loop}(x, yz)$ (for each x) in $O(n)$ time

⇒ $O(n^2 \log n)$ time $\quad [O(n^2)]$

Cabello Chambers E $O(g^2 n \log n)$ time

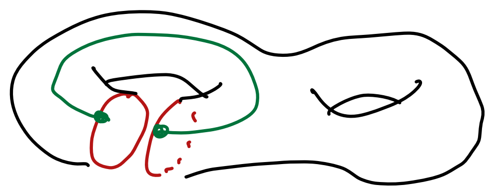
whp

$O(g^3 n \log n)$ or $O(g^2 n \log^2 n)$



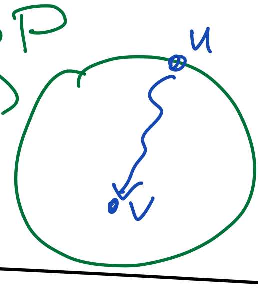
we can find collection C of cycles such that

1. any non-sep cycle crosses atleast one cycle in C
2. shortest non-sep cycle σ crosses some cycle in C exactly once.
3.



MSSP

$O(g^2 n \log n)$



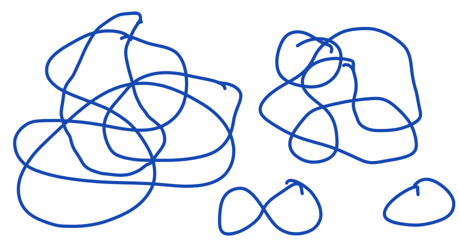
Cycles + Boundaries

Surface map Σ n vertices   Euler genus $\bar{g}$ (T, L, C)

Even subgraphs

= union of edge-disjoint cycles

= sym. diff of cycles.



Cycle space $Z_1(\Sigma)$ generated by simple cycles, $\oplus$

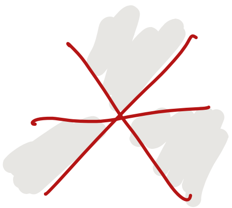
Fundamental cycles $\{\text{cycle}_T(e) \mid e \notin T\}$

$$Z_1(\Sigma) = \mathbb{Z}_2^{E-(V-1)}$$

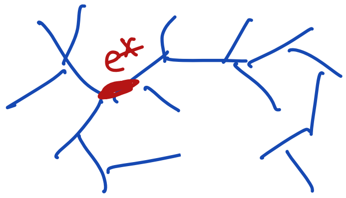
$$\boxed{H = \bigoplus_{e \in H \setminus T} \text{cycle}_T(e)}$$

Boundary subgraph = boundary of partition of faces.

every bdry is even



Boundary space $B_1(\Sigma) \le Z_1(\Sigma)$



Any edge $e \in C$
splits C^* into two trees.
↳ partition of faces of Σ

Fundamental. $\mathrm{bdry}_C(e)$ = boundary of that partition

$$\boxed{B = \bigoplus_{e \in B \cap C} \mathrm{bdry}_C(e)}$$

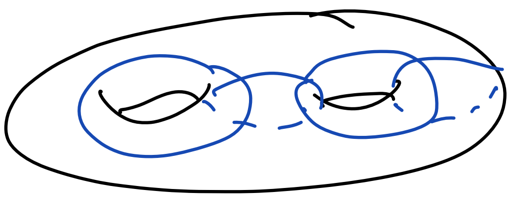
$$B_1(\Sigma) = \mathbb{Z}_2^{F-1}$$

Two even subgraphs H and H' are homologous
if $H \oplus H'$ is a boundary.

$$\begin{aligned} H_1(\Sigma) &= Z_1(\Sigma)/B_1(\Sigma) \\ &= \mathbb{Z}_2^{E-(V-1)} / \mathbb{Z}_2^{F-1} \\ &\cong \mathbb{Z}_2^{E-(V-1)-(F-1)} \\ &\cong \mathbb{Z}_2^{E-|T|-|C|} = \mathbb{Z}_2^{|L|} = \mathbb{Z}_2^{g} \end{aligned}$$

$$\boxed{H = \bigoplus_{e \in H \cap C} \mathrm{bdry}_C(e) \oplus \bigoplus_{i \in I} \mathrm{cycle}_T(\ell_i)}$$

$L = \{\ell_1 \ldots \ell_g\}$



These fundamental cycles form a homology basis