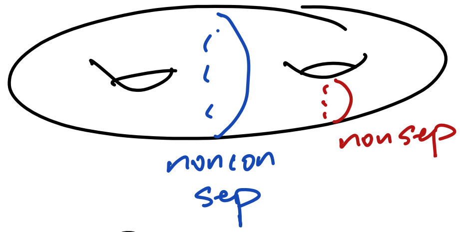
ICES Forms — due next Thu/Wed
Presentation scheduling form soon.
Reports due in 1 week(ish)

Shortest interesting cycles



$O(n^2 \log n)$ time $\longrightarrow$ $O(g^2 n \log n)$

Detour: Homology

"Cycles" = Even subgraphs $\quad \mathbb{Z}_2^{E-V+1} = Z(\Sigma)$

Boundary subgraphs $\quad \mathbb{Z}_2^{F-1} = B(\Sigma)$

Two even subgraphs H and H' are homologous iff $H \oplus H'$ is boundary

Homology classes $\quad H_1(\Sigma) = Z(\Sigma)/B(\Sigma)$

$$= \mathbb{Z}_2^{E-(V-1)-(F-1)}$$

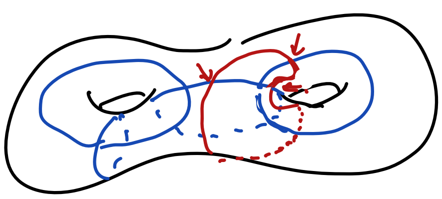
$$= \mathbb{Z}_2^{\overline{g}} = \mathbb{Z}_2^{\ell}$$

System of cycles:

Pick (T, L, C)

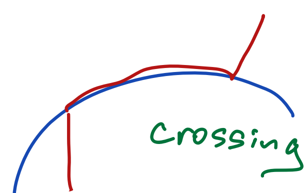
$C = \{\text{cycle}_T(e) \mid e \in L\}$

Characterizing homology



$cr(\gamma) = (0, 0, 1, 0)$

Crossing vectors



$cr(\alpha, \beta)$ = # times α and β cross mod 2

α boundary $\Rightarrow cr(\alpha,\beta)=0$

$cr(\alpha,\beta)=1 \Rightarrow \alpha$ and β are not boundaries

$cr(\alpha\oplus\beta,\gamma) = cr(\alpha,\gamma)\oplus cr(\beta,\gamma)$ ← This is consistent

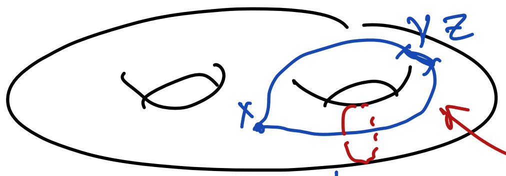
If H and H' are homologous $\Rightarrow cr(H,\gamma)=cr(H',\gamma)$

homology-invariant

H is <u>null</u>-homologous $\Leftrightarrow cr(H,\gamma)=0$
for every γ in system of cycles.

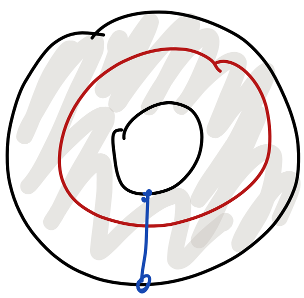
<u>Shortest non-sep cycles</u>

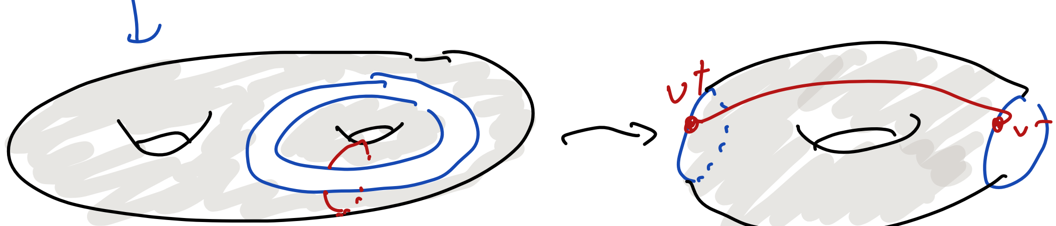
(T, L, C) — T = shortest-path tree



Shortest nonsep cycle σ
crosses this <u>at most</u> twice

$\Rightarrow \sigma$ crosses some cycle(e) for $e\in L$
<u>exactly</u> once.





MSSP

<u>Co</u>homology = dual homology

system of cocycles $\{cycle^*_{C^*}(e^*) \mid e\in L\}$

<u>Homology annotation</u>

$[e]$ = vector of $\bar{g}$ bits

$[e]_i = 1$ iff $e\in cocycle_C(\ell_i)$

$L=\{\ell_1,\ell_2\ldots\ell_{\bar{g}}\}$

COnvariant
COordinate
COmpute

$$[H] = \bigoplus_{e \in H} [e] \in \mathbb{Z}_2^{g}$$

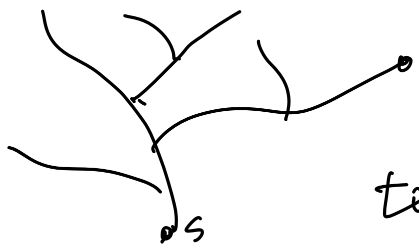
$$[H \oplus H'] = [H] \oplus [H']$$

Lemma: $[\text{cycle}_T(\ell_i)]_j = 1$ iff $i = j$

Thm: H and H' are homologous $\Leftrightarrow$ $[H] = [H']$

Cor: H is boundary iff $[H] = 0$

we can determine if $[H] = 0$ in $O(gk)$ time where $k = \#E[H]$



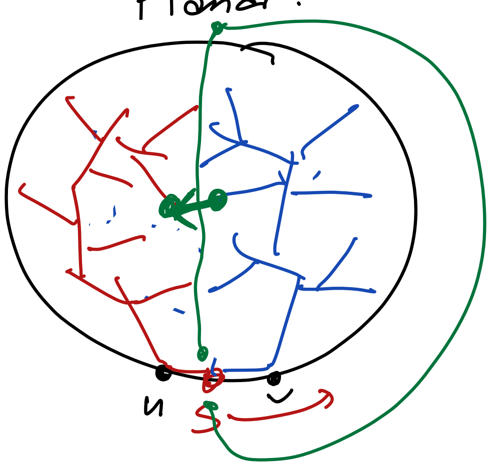
$[\text{path}(s,v)]$

total $O(gn)$ time

MSSP

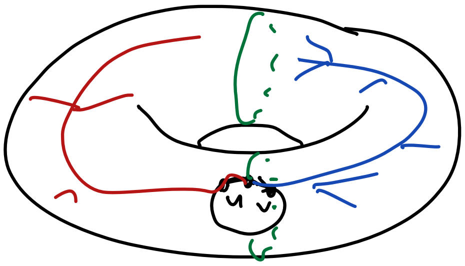
MSSP by parametric shortest path [Cabello Chambers E]

Planar:

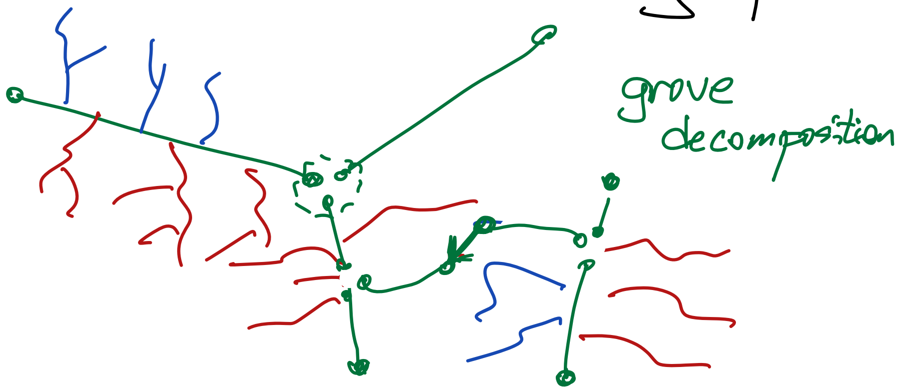


1. All active darts on one path in Σ^* (cycle)
2. We can find next pivot $O(\log n)$. Execute each pivot $O(\log n)$ using dynamic forest DSs.
3. Each dart pivots into T at most once. $\Rightarrow O(n)$ pivots

$\Rightarrow O(n \log n)$ time



(1) Active darts lie on cut paths of a reduced dual cut graph

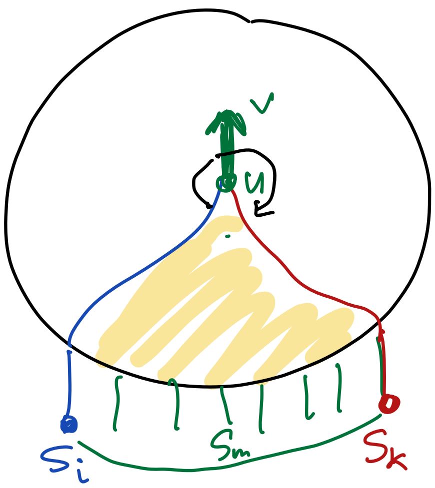


(2) Find next pivot in $O(g \log n)$ time
execute in $O(\log n)$ time

(4) Each dart pivots into T at most $g+1$ times

$\Rightarrow O(g^2 n \log n)$ time

MSSP by recursive contraction [Das et al 2021]



Compute sh. path trees at s_i and s_j $O(n \log n)$

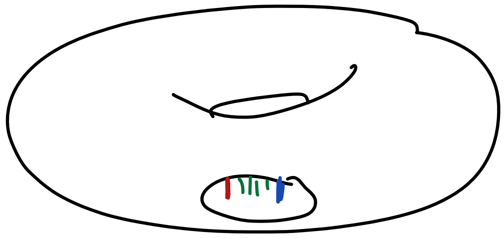
If $u \to v$ is properly shared by T_i and T_k
also T_j for all $i \leq j \leq k$
$\Rightarrow$ contract $u \to v$

(3) Each dart $u \to v$ is prop. shared by one interval of sources.

$\Rightarrow$ At any level of recursion tree total # verts is $O(n)$ $\leftarrow O(n \log n)$

$\Rightarrow O(n \log^2 n)$

Surfaces:



Shortest paths:
$O(n \log n + gn)$

(3) $O(n) \to O(gn)$

(1) $u \to v$ property shared by T_i and T_k iff

- $u = pred_i(v) = pred_k(v)$
- $pred_i(u) \to u$, $u \to v$, $pred_k(u) \to u$ } in cw order around u
- $[path_i(v)] = [path_k(v)]$

Total time in each subproblem $= O(gn + n \log n)$

Overall: $O((g + \log n)\, gn \log n)$

$= O(g^2 n \log n)$