Project Logistics

- Reports due next Wednesday (ish)
- Talks during Finals week
  Tentatively Mon+Tue
  Watch for email about signup form later today

ICES closes next Thursday 5pm

★ **Minimum Cuts — today**

Planar: $O(n \log\log n)$

$O(n)$ unit cap

$2^{O(g)} n \log n$

↑ $128^g = 2^{7g}$

$m^{1+o(1)}$

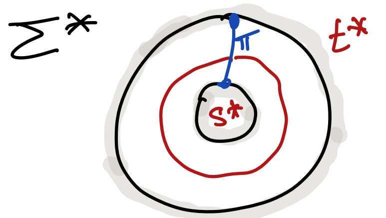
Maximum Flows — Wed

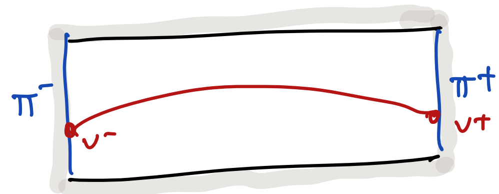
Planar: $O(n \log n)$

$O(n)$ unit cap

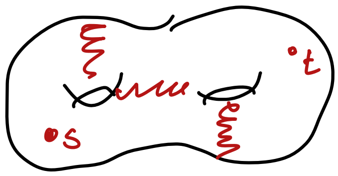
$O(g^4 n \log^2 n \log^2 C)$

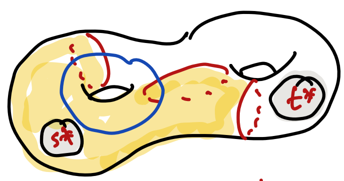
int. capacities





MSSP





≤ g+1 cycles

⇓

Min (s,t) cut in Σ

Min wt subgraph in Σ* homologous with ∂s*

⇓ ⇑ DP

Min wt cycle in Σ* in every nontrivial homology class.

→ MSSP in the homology covering space

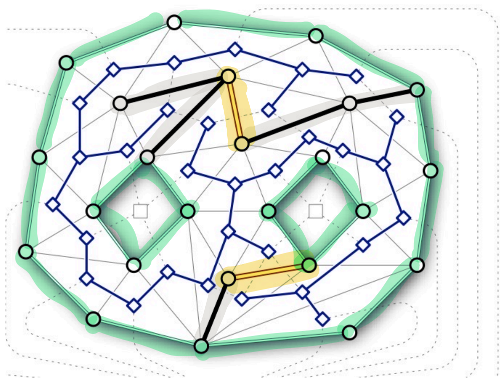
Surface map with boundary	Punctured surface map
Missing faces	**Missing vertices**
Missing open subsets	Missing closed subsets

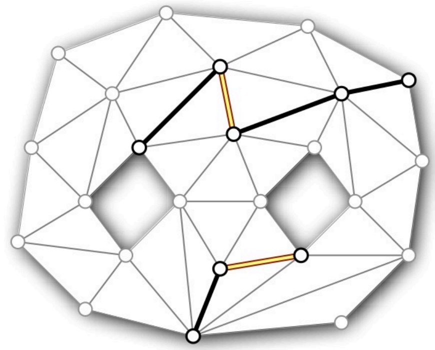
$\longleftrightarrow$

boundary edges ↓ Forest – hits every int vertex – each tree has one bdry vertex ↓ dual spanning tree ↙

$(\partial\Sigma, F, C, L)$

Forest-cotree decomposition





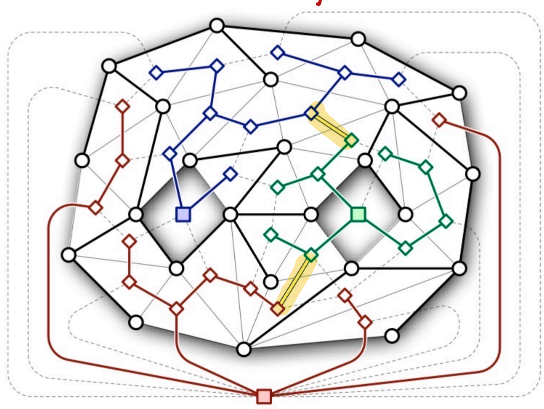
system of arcs

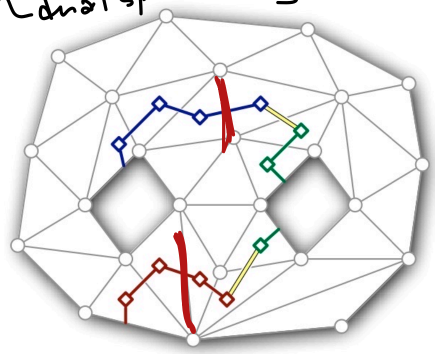
spanning tree

(T, F, L)

dual spanning forest, one component per missing face

Tree-coforest decomposition

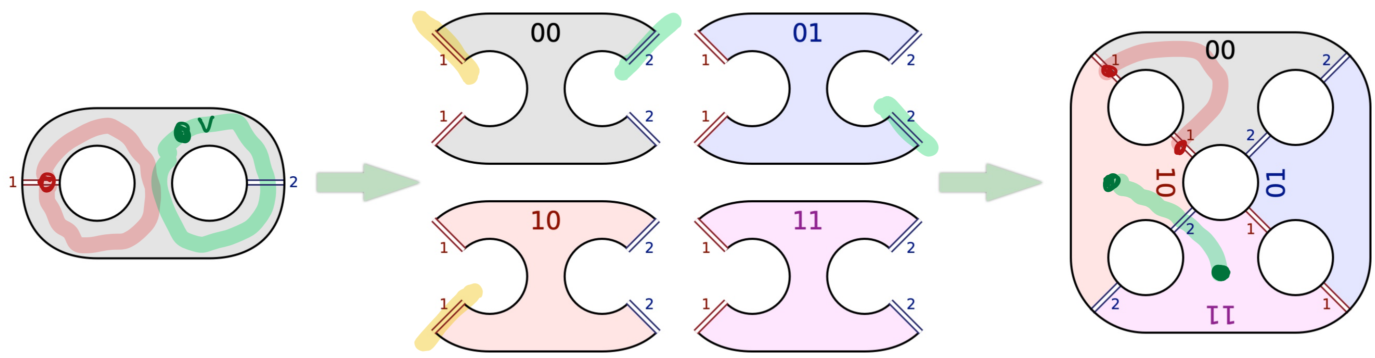




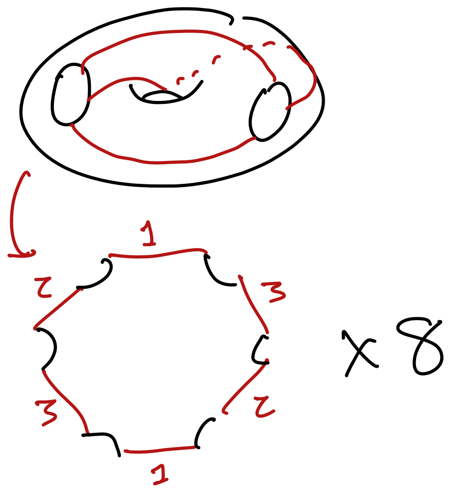
system of "cuarcs"

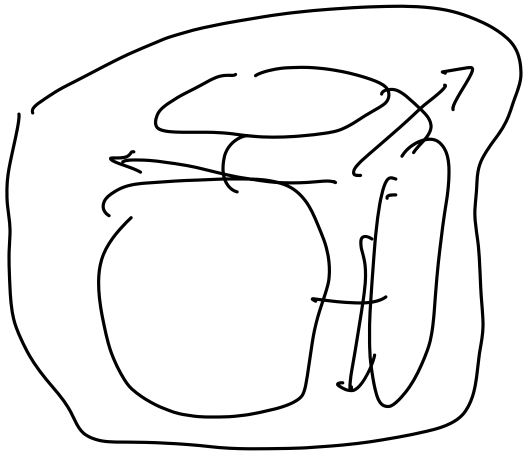
define homology signatures $[e]$

Prep: Build both greedy FCD (shortest path tree)
and TCFD (homology sigs)



$\mathbb{Z}_2$-Homology cover





$\bar{\Sigma} = (\bar{V}, \bar{E}, \bar{F})$ $\qquad \bar{V} = V \times \{0,1\}^{\beta}$ $\qquad \beta = g+1$

$\bar{E} = \{(u,h).(v, h \oplus [uv]) \mid uv \in E, \ h \in \{0,1\}^{\beta}\}$

$\bar{F} = \{(F,h) \mid \ldots\}$

"voltage construction"

Lemma: cycle γ in Σ thru vertex v

↓ ↑

path from (v,h) to $(v, h \oplus [\gamma])$

Shortest cycle in Σ thru v in homology class h

↓ ↑

Shortest path in $\bar{\Sigma}$ from $(v,0)$ to (v,h)

↳ crosses any other shortest path in $\bar{\Sigma}$ at most once

① Build greedy system of arcs $\alpha_1 \dots \alpha_\beta$

↑ 2 shortest paths

Lift to arcs $\bar{\alpha}_1 \dots \bar{\alpha}_\beta$ in $\bar{\Sigma}$

For any $h \neq 0$, shortest path $(v,0)$ to (v,h) must cross some $\bar{\alpha}_i$

For each index i

- slice $\bar{\Sigma}$ along $\bar{\alpha}_i$
  - for each vertex $v \in \alpha_i$
    - find shortest path from (v^-, h^-) to $(v^+, h^- \oplus h)$ in $\bar{\Sigma} \setminus \bar{\alpha}_i$

$2^{O(g)}$ instances of MSSP

in a surface map with genus $2^{O(g)} = G$

$N = 2^{O(g)} n$ verts + edges

$O(G^2 N \log N)$

$= 2^{O(g)} n \log n$

shortest cycles in every hom. class

↓

Shortest subgraphs in every hom. class.

> Shortest(h, k) = min wt subgraph in hom class h consisting of $\leq k$ cycles

Shortest(h, 1) given

$$\text{Shortest}(h, k) = \min\{\text{Shortest}(h_1, k-1) + \text{Shortest}(h_2, 1) \mid h_1 \oplus h_2 = h\}$$

$\text{Shortest}(h, \bar{g}+1)$ in $2^{O(g)}$ time.

$O(4^{\beta}\beta)$

$O(g^c \, n \log n)$ time?

$g^{O(g)} \, n \log\log n$ ✓