Presentations — Fill out schedule form by tomorrow (Thu)
Reports "due" today 11:59pm
20 min.

Minimum Cuts

$\mathbb{Z}_2$ – homology

covering space

optimal cycles/paths

MSSP

$2^{O(g)} n \log n$ time

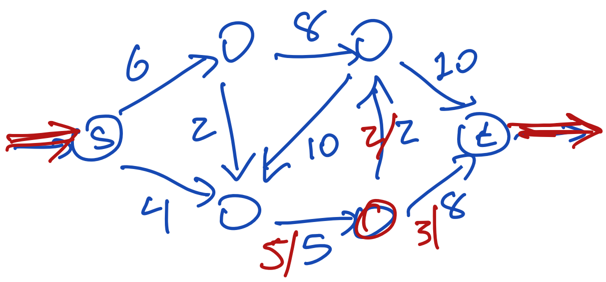
Maximum Flows

$\mathbb{R}$ – homology ⇐

shortest paths

linear programming in $O(g)$ variables

ellipsoid method



value of flow F

$$= \sum_{s \to v} F(s \to v)$$

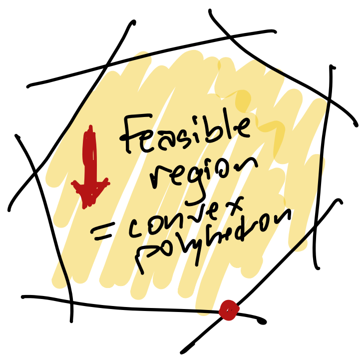
$$F(u \to v) = -F(v \to u)$$

$$\boxed{\begin{array}{ll} & \max \sum_{s \to v} F(s \to v) \\ \text{s.t.} & \sum_{u \to v} F(u \to v) = 0 \quad \text{for all } u \neq s, t \\ & F(u \to v) \le c(u \to v) \end{array}}$$

E variables

$V-2+2E$ constraints

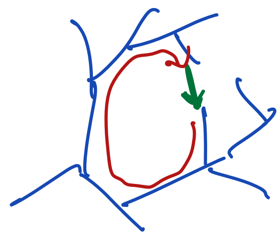
flow space $= \mathbb{R}^E$



$\mathbb{Z}_2$ - homology → $\mathbb{R}$ – homology

even subgraphs ⟶ circulations = sums of cycles

boundary subgraphs ⟶ boundary circulation = sum of face boundaries

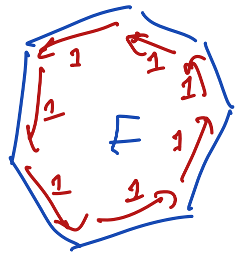


cycle basis

= Fundamental cycles wrt any sp. tree.

cycle space $Z_1(\Sigma) = \mathbb{R}^{E-V-1}$

∂f = one unit of flow ccw around boundary of f



Face potential $\alpha : F \to \mathbb{R}$

$\partial\alpha : D \to \mathbb{R}$

$\partial\alpha(d) = \boxed{\alpha(\mathrm{left}(d)) - \alpha(\mathrm{right}(d))}$

boundary space $\mathbb{R}^{F-1}$

(T, L, C) — name bdry circ by values of darts in C

Two circs ϕ ϕ' are homologous if $\phi - \phi' = \partial\alpha$ for some α

$H_1(\Sigma) = \mathbb{R}^{2g}$

~~Planar~~ Surface setting: There is a feasible boundary circulation in Σ

$\Longleftrightarrow$

There is no negative cycle in Σ^*

Feasible boundary circ in Σ $\Longleftarrow$ shortest paths in Σ^*

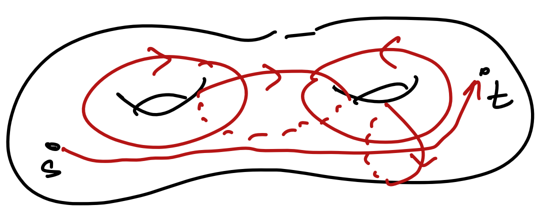
$\alpha(f)$ $\Longleftarrow$ $\mathrm{dist}(f^*)$

every circ is a bdry circ.

Corollary: For any flow f in Σ,
There is a feasible flow homologous with f
iff
Σ^*_f has no negative cycles

f and f' are homologous iff $f - f'$ is a bdry circulation

Flow homology space $= \mathbb{R}^{2g+1}$



Every flow is homologous with wt sum of path + 2g cycles

Given F we can find feasible F' homologous to F (if it exists) by computing shortest paths in Σ^*_F

Planar : $O(n \log^2 n) \to O(n \log^2 n / \log\log n)$

Surface : $O(n \log^2 n / \log\log n)$ start with use nice $O(\frac{n}{g})$-division instead of cycle sep at top level

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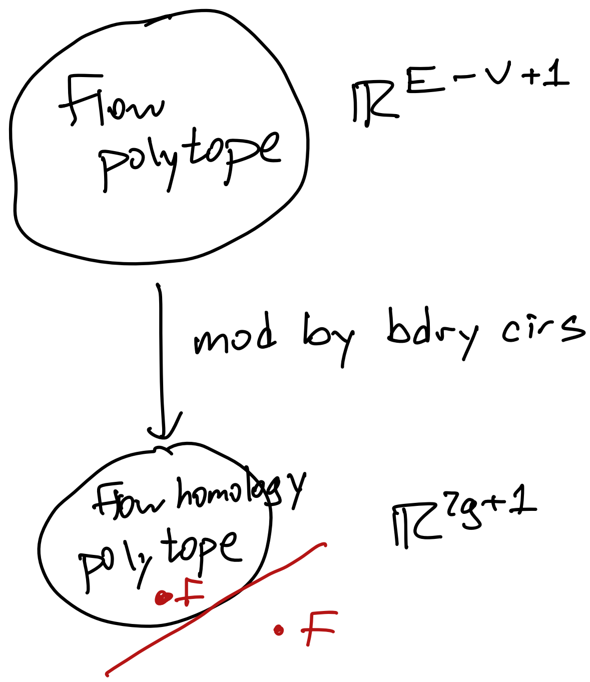


$O(n)$ constraints

$n^{O(g)}$ constraints

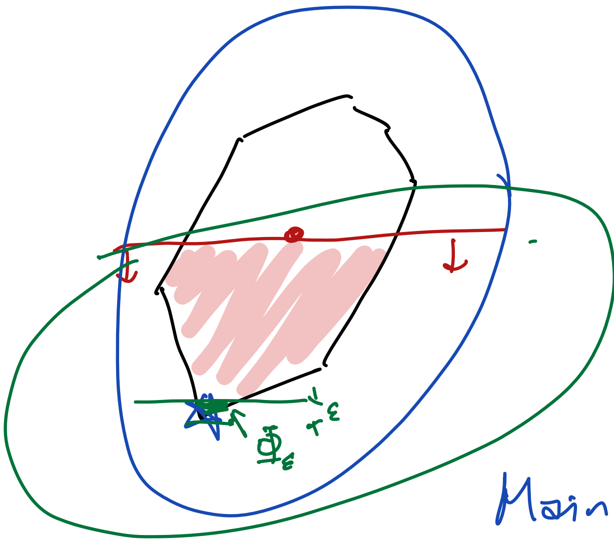
We have a membership + separation oracle

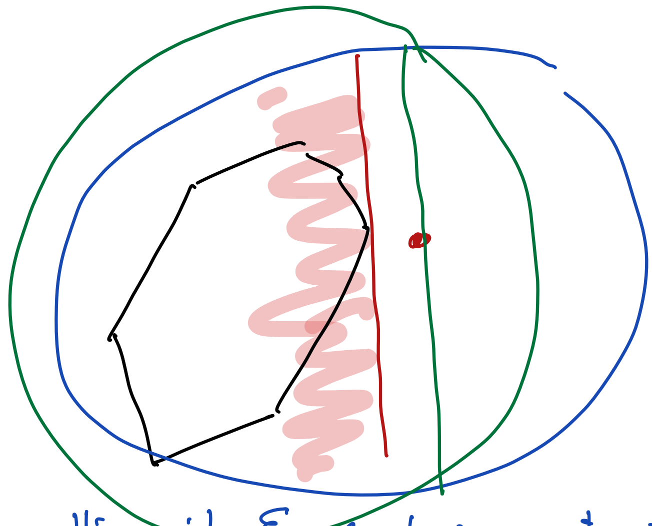
Flow homology LP has $2g+1$ variables too many constraints so we must solve it implicitly.

<u>Ellipsoid method</u>

1. all coeffs are in $\mathbb{Z}$ capacities
2. Initial $\mathcal{E}_0$
3. # iterations $= O(\log(\mathrm{vol}\,\mathcal{E}_0 / \mathrm{vol}\,\Phi_\varepsilon))$

Ellipsoid





Maintain ellipsoid $\mathcal{E}$ contains opt pt
Query centroid of $\mathcal{E}$.
$\rightarrow$ cuts $\mathcal{E}$ thru center
$\mathcal{E} \leftarrow$ min. ellipsoid (half of $\mathcal{E}$)

After every $O(d)$ iteration
vol $\mathcal{E}$ drops by factor of 2.

#iterations $= O(g^2 \log C)$ $\quad$ C = sum of capacities
for each: $O(n \log^2 n)$
$\times O(g^2 \log C)$ precision

$$\boxed{O(g^4 n \log^2 n \log^2 C) \text{ time}}$$

Open problem: edge ~~disjoint~~ disjoint s.t paths on torus
in $O(n \operatorname{polylog} n)$ time?
combinatorial alg.